THE INVERSE PROBLEM OF VERY INSTANTANEOUS PLATE KINEMATICS BY USING VLBI DATA

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IBARAKI 314

Introduction

In order to determine the present parameters of the relative motions of the lithospheric plates, ocean magnetic anomalies, transform fault azimuths and earthquake slip vectors have been used (Minster et al., 1974; Minster & Jordan, 1978). Among them, ocean magnetic anomaly data provide information on the speeds of the ocean floor spreading at mid-oceanic ridges while the others provide information on the directions of the relative plate motions at the plate boundaries. Minster et al. (1974) combined these data of different types and estimated the rotation pole positions and the rotation rates for individual plates using the least squares fitting.

Although these three data sets are equally used to obtain the "instantaneous" plate motion parameters, time spans represented by these data sets are not same. Minster & Jordan (1978) averaged the time span of the last 3Ma to obtain the present spreading rates. However, earthquake slip vectors represent much shorter time span, say the last ten or twenty years. Also transform fault azimuths do not necessarily represent truely present-day relative plate motion direction. Transform faults are a kind of "geological" structure developed during the geological time and transform faults themselves are known to alter into other types of plate boundaries such as spreading centers (leaky transform faults) or subduction zones (transform faults with slight thrust component) in response to the sudden change of the relative plate movements. In a strict sense, only earthquake slip vectors are considered to represent today's plate motion.

Recent developments of various techniques enabled the very precise geodetical measurements between two remote points which belong to different plates. Centimeter accuracy of the measurement of the distance of such points yields the potential of clarifying the relative movements between these points which originate from the plate motion. Among such recently developed techniques, very long baseline interferometry (VLBI) has been suggested to be the most promising technique for geodynamics (e.g., Coates et al., 1975). In 1984, Radio Research Laboratories (RRL) succeeded in measuring the distances between Kashima station and American stations with an accuracy of a couple of centimeters. During the next five years, annual or semiannual VLBI experimemts between several American, Pacific, European and Japanese stations are programed. The successful results of this project will greatly contribute to elucidate the true process of the instantaneous plate tectonics.

In this short article, I propose the changing rates of the baseline lengths among points of various plates as a possible substitute of the geophysical/geological data conventionally used to determine instantaneous plate motions. I also present algorithms of the "forward" problem and the "inverse" problem between the changing rates of interplate baseline lengths and plate tectonic parameters.

Forward problem

As the first step, let us consider how large is the changing rate of the interplate baseline length. Now, two VLBI stations A and B are assumed to be located on two different plates i and j, respectively. I express the positions of the two stations by the geocentric Cartesian coordinates as A (x_a, y_a, z_a) and B (x_b, y_b, z_b). Then the length of the baseline (i.e., the distance) between A and B should be expressed as

$$D_{BA}{}^2 = (x_a - x_b)^2 + (y_a - y_b)^2 + (z_a - z_b)^2. \qquad (1)$$

The changing rate of the baseline length is obtained as the time derivative, that is,

$$\dot{D}_{BA} = (1/D_{BA}) \{ (x_a - x_b)(\dot{x}_a - \dot{x}_b) + (y_a - y_b)(\dot{y}_a - \dot{y}_b) + (z_a - z_b)(\dot{z}_a - \dot{z}_b) \}, \qquad (2)$$

where dots represent the time derivatives.

In order to obtain the relationship between the changing rate and the plate tectonic parameters, it is convenient to express (x_a, y_a, z_a) and (x_b, y_b, z_b) by using the latitudes and longitudes. Let θ_a and ϕ_a be the latitude and longitude of the station A and θ_b and ϕ_b those of the station B, respectively, then the three components are written as

$$\begin{pmatrix} x_a \\ y_a \\ z_a \end{pmatrix} = \begin{pmatrix} R\cos\theta_a\cos\phi_a \\ R\cos\theta_a\sin\phi_a \\ R\sin\theta_a \end{pmatrix} \qquad \begin{pmatrix} x_b \\ y_b \\ z_b \end{pmatrix} = \begin{pmatrix} R\cos\theta_b\cos\phi_b \\ R\cos\theta_b\sin\phi_b \\ R\sin\theta_b \end{pmatrix}, \qquad (3)$$

where R denotes the earth's radius. It is widely known that the instantaneous motion of a rigid plate lying on a spherical surface can be completely and uniquely described in terms of the rotation around an Euler pole, which makes the velocity field of m plates written by specifying the m instantaneous angular velocity vectors of these plates. Then the instantaneous velocity vector of the points A/B belonging to the plates i/j are given as the outer products of the angular velocity vectors of the plates and the position vectors of A/B. Now if we suppose the rotation pole of the plates i and j at latutudes of θ_i, θ_j and longitudes of ϕ_i, ϕ_j, with the rotation rates of ω_i and ω_j, then the velocity vector of the stations A and B, that is, ($\dot{x}_a$, $\dot{y}_a$, $\dot{z}_a$) and ($\dot{x}_b$, $\dot{y}_b$, $\dot{z}_b$) are expressed as follows;

$$\begin{pmatrix} \dot{x}_a \\ \dot{y}_a \\ \dot{z}_a \end{pmatrix} = \begin{pmatrix} R\,\omega_i(\sin\theta_a\cos\theta_i\sin\phi_i - \cos\theta_a\sin\phi_a\sin\phi_i) \\ R\,\omega_i(\cos\theta_a\cos\phi_a\sin\theta_i - \sin\theta_a\cos\theta_i\cos\phi_i) \\ R\,\omega_i\cos\theta_a\cos\theta_i\sin(\phi_a - \phi_i) \end{pmatrix} \qquad (4a)$$

$$\begin{pmatrix} \dot{x}_b \\ \dot{y}_b \\ \dot{z}_b \end{pmatrix} = \begin{pmatrix} R\,\omega_j\,(\sin\theta_b\cos\theta_j\sin\phi_j - \cos\theta_b\sin\phi_b\sin\phi_j) \\ R\,\omega_j\,(\cos\theta_b\cos\phi_b\sin\theta_j - \sin\theta_b\cos\theta_j\cos\phi_j) \\ R\,\omega_j\cos\theta_b\cos\theta_j\sin(\phi_b - \phi_j) \end{pmatrix} \quad (4b)$$

From the equations (2), (3) and (4), we obtain the changing rate of the baseline length as a function of the latitudes/longitudes of the stations and the plate motion parameters, i.e., the rotation pole positions (latitudes and longitudes) and the rotation rates of the plates i and j, that is,

$$\dot{D}_{BA} = (R^2/D_{BA})\,[\cos\theta_a\cos\theta_b\sin(\phi_a-\phi_b)(\omega_i\sin\theta_i-\omega_j\sin\theta_j)$$
$$- \cos\theta_a\sin\theta_b\,\{\omega_i\cos\theta_i\sin(\phi_a-\phi_i) - \omega_j\cos\theta_j\sin(\phi_a-\phi_j)\} +$$
$$\sin\theta_a\cos\theta_b\{\omega_i\cos\theta_i\sin(\phi_b-\phi_i) - \omega_j\cos\theta_j\sin(\phi_b-\phi_j)\}\,]\,. \quad (5)$$

This equation gives a solution to the forward problem: given the parameters of relative plate motions, we can calculate the changing rate of the interplate baseline length between any stations.

Inverse problem

Now let us consider the inverse problem of instantaneous plate kinematics: given a number of observations of the changing rates of the interplate baseline lengths, what is the best representation of instantaneous plate motions? This problem is very similar to the inverse problem described in Minster et al. (1974) and the description in this section is analogous to their discussion.

Suppose the observations of the changing rates of the baseline lengths were obtained for n interplate baselines, then these values constitute a vector $\vec{d}^0$ made of n components. Similarly, the 3m-3 independent components of a plate motion model can be serially arranged to form a model vector $\vec{m}$:

$$\vec{m} = (\theta_1, \phi_1, \omega_1, \ldots\ldots, \theta_{m-1}, \phi_{m-1}, \omega_{m-1}), \quad (6)$$

where the reference frame was arbitrarily chosen so that the m-th plate is fixed because only $M = 3m - 3$ components are independent for a relative motion model.

For any $\vec{m}$ we can compute the vector $\vec{d}(\vec{m})$ using the equation (5). If the data were error free and the rigid plate model was correct, the representation we seek would satisfy the equation

$$\vec{d}(\vec{m}) = \vec{d}^0. \quad (7)$$

However the observations are not perfectly compatible and are contaminated with errors. If we suppose the changing rate observations of the individual baselines are statistically independent and free of bias, their fundamental distribution are thought to be Gaussian. Hence, to obtain a model representation, one might maximize a likelihood function proportional to

$$\exp\left\{-\sum_{i=1}^{n}\frac{[d_i^{0}-d_i(\vec{m})]^2}{2\sigma_i^2}\right\} \tag{8}$$

where d_i^{0} is the i-th component of $\vec{d}^{0}$, $d_i(\vec{m})$ is the value of the i-th data function evaluated at $\vec{m}$, σ_i^2 (i=],....,n) the variance of the i-th rate datum. A necessary and sufficient condition for the likelihood to be maximized is that the function

$$F = \sum_{i=1}^{n}\frac{[d_i^{0}-d_i(\vec{m})]^2}{2\sigma_i^2} \tag{9}$$

be minimized. In other words, the problem is reduced to what we call non-linear least squares fitting and the best fitting model can be calculated by an iterative refinement of the parameters using the Gauss-Newton method. Ready-made standard software of least squares fitting are now widely equipped with large computer centers (e.g., Statistical Analysis by Least Squares fitting: SALS; Nakagawa and Oyanagi, 1982) and all we have to do is to calculate the elements of the Jacobian matrix.

The Jacobian matrix has the dimension of n x m and consists of the elements made of the derivatives of one of the data with respect to one of the model parameters, that is, the (i, k) elememt is given as the derivative of the i-th datum with respect to the k-th parameter. In the present case, they are given as follows;

$$\begin{aligned}\partial\dot{D}_{BA}/\partial\theta_k=(\delta_{ik}-\delta_{jk})R^2\,\omega_k\,\{&\cos\theta_a\cos\theta_b\sin(\phi_a-\phi_b)\\ &+\cos\theta_a\sin\theta_b\sin\theta_k\sin(\phi_a-\phi_k)\\ &-\sin\theta_a\cos\theta_b\sin\theta_k\sin(\phi_b-\phi_k)\}/D_{BA}\end{aligned} \tag{10a}$$

$$\begin{aligned}\partial\dot{D}_{BA}/\partial\phi_k=(\delta_{ik}-\delta_{jk})R^2\,\omega_k\,\{&\cos\theta_a\sin\theta_b\cos\theta_k\cos(\phi_a-\phi_k)\\ &-\sin\theta_a\cos\theta_b\cos\theta_k\cos(\phi_b-\phi_k)\}/D_{BA}\end{aligned} \tag{10b}$$

$$\begin{aligned}\partial\dot{D}_{BA}/\partial\omega_k=(\delta_{ik}-\delta_{jk})R^2\,\{&\cos\theta_a\cos\theta_b\sin(\phi_a-\phi_b)\sin\theta_k\\ &-\cos\theta_a\sin\theta_b\cos\theta_k\sin(\phi_a-\phi_k)\\ &+\sin\theta_a\cos\theta_b\cos\theta_k\sin(\phi_b-\phi_k)\}/D_{BA},\end{aligned} \tag{10c}$$

where δ_{ik} and δ_{jk} are Kronecker deltas.

Actual calculations were done by using standard software for least squares fitting, SALS, by using dummy data and the parameters were observed to converge to the expected values without any troubles.

Discussion and summary

In order to determine plate tectonic parameters, we need only minimum number of relative motion data. However, when we have more than the miminum amount of data, it becomes possible to calculate all the relative motions of plates simultaneously and examine the self-consistency of the assumption of plate tectonics such as the rigidity of the plates and constant area of the earth. Chase (1972), Minster et al. (1974), Minster & Jordan (1978) succeeded in obtaining such plate tectonic parameters which are consistent with all the relative motion data representing the last several millions of years. However, there is no guarantee that the plates are behaving as rigid bodies in the context of instantaneous movements. Now, our major concern is how the plates are behaving today, in other words, wheather the theories of plate tectonics are applicable or not for such a short time span as a few years.

There are several possibilities of the instantaneous plate kinematics which can be imagined. One plausible possibility is that the plates are moving in a similar fashion to the already known time-averaged relative plate motion. In this case, the plate tectonic parameters will converge to the same values as the currently available parameters such as RM-2 of Minster & Jordan (1978). Another possibility is that the parameters converge to certain values which are much different from RM-2. In this case, plates are behaving as rigid bodies but their rotation rates and rotation axes are envisaged to be different from time-averaged ones. There is also a possibility that the parameters do not converge at all and in this case the model assumption of rigid plates are considered to be wrong.

Now the world-wide VLBI network is not so sufficient as to establish the present day relative kinematics for all the known plates, but at least it would be possible to examine the plate tectonic theories over the several plates where multiple VLBI stations are available. The future development of the "mobile" VLBI stations may enable the measurements over the truely world-wide network covering all the known plates and clarify the instantaneous kinematics of these plates.

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