Development and Application of New Techniques for Surface Wave Tomography

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Statement

All the works presented in this thesis are solely based on the original work of the author, unless otherwise mentioned in the text and the acknowledgment.

Kazunori Yoshizawa

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Abstract

We propose a new three-stage approach for surface wave tomography working with new techniques for surface wave analysis: multi-mode dispersion measurements using nonlinear waveform inversion and an efficient method to estimate the influence zone about a surface wave path. These topics are of importance for extending current methods of surface wave tomography that are commonly based on geometrical ray theory and on the approximation of great-circle propagation.

A method of fully nonlinear waveform inversion for a path-specific 1-D profile has been developed using a Neighbourhood Algorithm (NA). Unlike the traditional methods of waveform inversion which have been based on linearlised inversion techniques, the NA provides a means to find an ensemble of reasonably acceptable models without any calculations of derivatives with respect to model parameters. With different approaches to the parameterisation of the shear wavespeed profile, we can find models with significant differences in velocity variation with depth, which provide similar levels of fit to the observed waveforms. Although the models differ, the calculated phase dispersion for the first few modes of the surface waves are very close indeed. Therefore the 1-D models derived from the multi-mode waveform inversion can be regarded as an implicit summary of the path-specific dispersion for each of the modes.

We also examine an approximate zone of influence around the propagation path for a surface wave using a technique called Fresnel-area ray tracing (FRT) for surface waves. The influence zone about surface wave paths, over which surface waves are coherent in phase, is identified as approximately 1/3 of the width of the first Fresnel zone. The estimate of the influence zone can be efficiently calculated in laterally heterogeneous structure by using the FRT technique. This approach makes it possible to efficiently work with finite-width rays as well as deviations in propagation paths from the great-circle induced by moderate lateral heterogeneity as revealed by recent tomography models. Such finite-width rays should be of major benefit in enhancing ray-based surface wave tomography by taking into account the finite-frequency effects of surface wave propagation.

Utilising these techniques, a *three-stage inversion* scheme for surface wave tomography working with multi-mode phase dispersion maps as a function of frequency has been

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constructed, and then applied to the Australian region. The new approach provides a means of combining a variety of information, i.e., multi-mode dispersion, off-great-circle propagation and the finite-frequency effects, in a common framework.

Information of multi-mode dispersion can be extracted from any convenient methods, such as direct dispersion measurements for the fundamental mode, mode-stripping technique for higher modes, or using path-specific 1-D profiles as a summary of multi-mode dispersion. Multi-mode phase speed maps are obtained from ensembles of these observations using the conventional technique based on the great-circle approximation, and can then be iteratively updated considering ray-path bending and finite-frequency effects. A 3-D shear wavespeed structure is finally derived from these updated multi-mode phase speed maps by inverting local dispersion curves for local 1-D shear wavespeed models. In this approach, the smoothness of the model is controlled not only by the damping of the linearlised inversion and parameterisation of model space, but also by the influence zone, which makes it possible to introduce natural smoothing of velocity models based on physical constraints on surface wave propagation.

Several new 3-D models are derived from different types of inversions, i.e., with or without the effects of off-great-circle propagation and the influence zone. These models show a good agreement in large-scale velocity variations, whereas there are some differences in small-scale (a few hundred kilometers) features, mainly due to the effects of natural smoothing caused by the influence zone. More uniform horizontal resolution can be achieved by the introduction of the influence zone, although the apparent maximum resolution is somewhat reduced due to the effects of finite frequency of surface waves.

Although the influence zone is of great help to take account of the sampling region around surface wave paths in tomographic inversion, it does not encompass the entire region of scattering and diffraction around surface wave paths. More rigorous 2-D sensitivity kernels based on the Born and Rytov approximations can be constructed, working with WKBJ representation for surface wave potentials and paraxial ray theory. Such kernels depend on simple assumptions on a scalar-wave type approximation for surface waves, and require more computation than the influence zone. However, this type of broader kernels should be of importance in exploiting the full waveforms of surface waves by including more complex effects of surface wave scattering.

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Introduction

1

Probing into the Earth's interior using seismic waves is one of the major themes of seismology, and is essential to enhance our knowledge of the evolution of our planet and of the interactions between mantle dynamics and surface tectonics, which are closely connected to the occurrence of earthquakes and volcanic activity. One of the most remarkable jumps in understanding of the deep structure of the Earth was achieved through the development of seismic tomography beginning in the middle 1970's. Since then, seismologists have proposed various tomographic models for 3-D Earth structure.

The use of teleseismic body waves have revealed the detailed features of whole mantle; for example, lateral variations of discontinuities (e.g., Morelli & Dziewonski, 1987; Shearer & Masters, 1992) and whole mantle 3-D velocity structures (e.g., Dziewonski, 1984; Inoue et al., 1990; Tanimoto, 1990a; Su et al., 1994; Li & Romanowicz, 1996; van der Hilst et al., 1997; Widiyantoro et al., 1998; Kennett et al., 1998)

On the other hand, surface waves have been used to retrieve details of crustal and upper mantle structures for both regional and global scales in the form of 2-D phase and group speed structures (e.g., Trampert & Woodhouse, 1995; Ekström et al.,1997; Ritzwoller & Levshin, 1998) and 3-D shear wavespeed models (e.g., Woodhouse & Dziewonski, 1984; Zhang & Tanimoto, 1993a; van der Lee & Nolet, 1997; Debayle & Kennett, 2000a). Information from the Earth's normal modes are also helpful to retrieve long-wavelength features of the whole mantle structures (e.g., Resovsky & Ritzwoller, 1998, 1999; Ishii & Tromp, 1999, 2001).

Seismic surface waves are normally the most prominent phase in an observed seismogram from shallow and intermediate depth sources. This fact arises from the cylindrical-wave character of surface waves whose decay rate $(1/\sqrt{r}, \text{ where } r \text{ is distance from the source})$ is smaller than that of spherically spreading body waves (1/r). Although surface waves propagate two-dimensionally along the Earth's free surface, their energy reaches into the deeper part of the upper mantle, resulting in their dispersive character. Using the dispersion of surface waves, crustal and upper mantle structures have been extensively investigated by many researchers. In this thesis, we will start by describing the nature of surface waves, which are the most powerful tool to probe the 3-D upper mantle structure. Then, we propose a new approach for surface wave tomography using new styles of surface wave analysis, which can overcome the limitations in the conventional ray-based methods of surface wave inversion.

1.1 Surface wave ray theory

Most seismic tomography has been based on ray theory, which provides effective means to investigate the structure of the Earth. Asymptotic ray theory for surface waves in three dimensional structures with laterally slowly varying heterogeneity has been developed by Woodhouse (1974) and Babich et al. (1976), that is equivalent to WKBJ theory for surface waves (e.g., Dahlen & Tromp, 1998). Such analysis of surface waves is based on high-frequency approximations and is valid only in laterally smoothly varying media. Therefore strong velocity variations along a path cannot be treated with these methods.

Although ray theory relies on a high-frequency approximation, in case of the real Earth, ray theory is known to be a fairly good approximation for long-period surface waves, since they sample deeper part of the upper mantle where the lateral heterogeneity is not so large compared to the wavelength.

Woodhouse & Wong (1986) have shown that the perturbations of phase shift, arrival angle and amplitude anomalies can be represented by linearised integral equations along a ray. In order to extend the surface wave ray theory, Yomogida (1985) and Yomogida & Aki (1985) have developed the Gaussian beam methods for surface waves based on the asymptotic ray theory, and proposed a way to calculate surface wavefields at finite frequency. Surface wave WKBJ theory has been described in detail by Tromp & Dahlen (1992a,b).

These studies on surface wave ray theory have been the basis of the surface wave tomography. However, there are intrinsic limitations in such theories, which cannot treat scattering and finite frequency effects of surface wave propagation. Wang & Dahlen (1995a) have empirically obtained a condition for the validity of surface wave ray theory, with an assumption that the width of the first Fresnel zone must be much smaller than the scale length of lateral heterogeneity.

1.2 Surface wave tomography

Almost all surface-wave tomography models have been derived by using the "phase" information contained in waveforms based on surface-wave ray theory (e.g., Woodhouse, 1974; Woodhouse & Wong, 1986) with the approximation of propagation along the great circle.

Other possible types of data which can be used to retrieve the mantle structure based on ray theory are "polarization" and "amplitude" anomalies, which are sensitive to the gradients in seismic wavespeed. Polarization anomalies, which can be observed as arrival angle anomalies of rays, have been shown to have the capacity to retrieve smaller scale heterogeneity than phase data (Laske & Masters, 1996; Yoshizawa et al., 1999).

Attenuation tomography (Romanowicz, 1995) has been based on the analysis of surface wave amplitude, but depends on detailed knowledge of the velocity distribution. Amplitude anomalies are sensitive to the second derivatives of the velocity structure, so that finescale changes in the structures may be detected by introducing such a data set (Yomogida & Aki, 1987; Laske & Masters, 1996). However the analysis of amplitude anomalies still has some difficulties since the amplitude of observed waveforms are very sensitive to the local site effects, the calibration of seismometers, knowledge of source mechanisms, and focusing and defocusing effects.

1.2.1 Global studies

Global studies on the surface wave tomography have initiated in the early 1980's based on the simple geometrical ray theory. With the approximation of surface wave propagation along the great-circle, a three-dimensional shear wavespeed model was derived from a waveform inversion (Woodhouse & Dziewonski, 1984). By measuring dispersion of fundamental-mode surface waves, global group and phase speed maps have been proposed (e.g., Nakanish & Anderson, 1982, 1983, 1984). Such dispersion maps are further inverted for 3-D shear wavespeed models (Montagner, 1986; Nataf et al., 1986). Global anisotropy maps have also been investigated by using such dispersion measurements (Tanimoto & Anderson, 1985, Montagner & Tanimoto, 1990, 1991).

In the 1990's, the number of broadband stations dramatically increased and high quality three-component broad-band observations became available, which allows us the construction of high resolution phase speed models (Zhang & Tanimoto, 1993b; Trampert & Woodhouse, 1995, 1996; Zhang & Lay, 1996; Ekström et al., 1997). The highest resolution global model at present is expanded up to degree 40 in spherical harmonics (Fig 1.1). Although a number of global models have been proposed, they are very well correlated



Fig. 1.1. Rayleigh-wave phase speed map at 100 s (Ekström et al., 1997)

in the lower orders for long wavelength structures, showing dominant features of degree 2 (Masters et al., 1982, Su & Dziewonski, 1991).

Most global models have been based only on the dispersion measurements for the fundamental mode, because fundamental mode analysis is rather straightforward compared to the higher modes. Van Heijst & Woodhouse (1997) have proposed a mode-stripping technique to measure higher-mode phase speeds from a single seismogram and they have applied this technique to obtain global multi-mode phase speed maps (van Heijst & Woodhouse, 1999). Such an approach is helpful for improving the vertical resolution of the tomography models, although this style of mode-stripping can only be applied to paths with epicentral distances longer than 30 degrees (van Heijst & Woodhouse, 1999). There is significant overlap between the higher mode branches and the fundamental mode for shorter distances, which makes it difficult to separate the contributions from different modes in an observed seismogram .



Fig. 1.2. Ray path coverage (top) and shear wavespeed model at 150 km (Bottom) (Debayle & Kennett, 2000a).

1.2.2 Regional studies

Regional surface wave tomography usually uses a different approach to most global studies. A common procedure for regional surface-wave tomography is a two-stage method using waveform fitting, such as partitioned waveform inversion (hereafter referred to as PWI) of Nolet (1990) and the waveform inversion using secondary observables developed by Cara and Lévêque (1987, hereafter CL).

Both techniques are based on two separate steps. In the first step, waveforms are inverted for path-average 1-D models. Then, the assemblage of such path-specific information is further inverted for a final 3-D model.

PWI has been applied to many regions on the globe, such as Europe (Zielhuis & Nolet, 1994), the western Pacific (Lebedev et al., 1997), North America (van der Lee & Nolet, 1997; Frederiksen et al., 2001) and Australia (Zielhuis & van der Hilst, 1996; Simons et al., 1999). A systematic automation method of the PWI has been proposed by Lebedev (2000) and has been applied to the western Pacific and eastern Asian region.

CL methods have also applied to several regions in combination with a regionalised inversion technique of Montagner (1986) to derive a 3-D model. Debayle & Lévêque (1997) applied such a technique to investigate the upper mantle structure in the Indian Ocean. An automated procedure using this technique has been developed by Debayle (1999) and applied to the Australian region (Debayle & Kennett, 2000a) and northern Africa (Debayle, Lévêque & Cara, 2001).

Regional inversions utilise paths with short propagation distance with high density of ray coverage (e.g., Fig 1.2 a), and the high resolution models (e.g., Debayle & Kennett, 2000a) could resolve down to a scale of a few hundred kilometers (Fig 1.2 b), whilst the resolving power in the global models is around 1000 km at most. Thus, there are significant differences in the resolution between the global and regional models (Fig 1.3). Such differences are mainly due to the use of comparatively long paths for global studies, which results in smoothing out the small scale features by path-averaging over longer paths (e.g., Kawakatsu, 1983; Passier & Snieder, 1995).

1.3 Studies on surface wave propagation in complex media

In the existence of strong lateral heterogeneity, the effects from scattering and modecoupling cannot be ignored. Kennett (1984) proposed a method to treat the effects of mode-branch coupling for 2-D structure with strong heterogeneity along the path, by solving coupled differential equations. This technique has been the basis of many works



Fig. 1.3. Phase speed maps of the fundamental-mode Rayleigh waves at 100 seconds of a global model (top panel: Ekström et al, 1997) and a regional (bottom panel) model that is estimated from the 3-D shear velocity model of Debayle & Kennett (2000a).

on coupled-mode approaches in 2-D and 3-D structures (e.g., Maupin & Kennett, 1987; Maupin, 1988, 1992; Tromp, 1994; Kennett, 1998a).

First-order scattering theory for surface waves in 3-D structure (e.g., Snieder,1986; Snieder & Nolet, 1987) can also be useful to treat the effects of local strong heterogeneity. In order to take into account appropriate finite-frequency effects in tomography models, 2-D sensitivity kernels have been investigated by some researchers (e.g., Li & Tanimoto, 1993; Li & Romanowicz, 1995; Marquering & Snieder, 1995) considering the coupling between mode-branches.

Recently 3-D sensitivity kernels have been developed by using coupled surface-wave modes (Marquering et al., 1998, 1999), the first-order scattering of body waves (Dahlen et al., 2000) and normal mode theory (Zhao et al., 2000). Such 3-D kernels enable us to exploit direct inversion to a 3-D model taking into account finite-frequency effects, although most such techniques require much more computation than simple ray theoretic methods.

1.4 Limitations in surface wave tomography

Although the success of surface wave tomography based on ray theory reveals detailed 3-D structures in the upper mantle, there still remain fundamental problems.

In the high frequency approximation of ray theory, a ray does not have a width. However, the actual seismic waves along a ray path should be affected by a finite area around a ray due to the finite frequency of wave propagation.

Fresnel areas around a ray path (or the appropriate great-circle) can be used as a guide to estimate an approximate region around a ray path which has significant effects on observed waveforms at finite frequency. In conventional regional tomography using pathaverage 1-D models, however, it has not been simple to accommodate frequency- and mode-dependent Fresnel areas around a path.

In both global and regional tomography, most studies have adopted the approximation of surface wave propagation along the great-circle. This works quite well as long as the lateral heterogeneity is not very strong, for longer period models (say longer than 40 second). However, the great-circle approximation tends to break down for short period surface waves (less than 30 seconds), which are more sensitive to the structures in the crust and uppermost mantle where strong velocity variations exist. Recent high-resolution tomography models (e.g., Debayle & Kennett, 2000a) have shown existence of moderate heterogeneity with $\pm 10\%$ velocity variations, which almost reaches the limit of the validity of simple ray theory with the great-circle approximation. We, therefore, need to take the effects of off-great-circle propagation into account.

In existence of strong lateral variations of velocity structures, we will also need to think about the validity of the path-average 1-D models, which have been widely used in most regional tomography as intermediary information for a final 3-D model. Since waveform inversions for a path-average 1-D model are, in general, based on the perturbation theory, we cannot allow large deviations in velocity structures from a reference model.

Hiyoshi (2001) has extensively investigated the capacity for model recovery for two types of waveform inversion methods using a set of synthetic tests, and has found that linearised inversion for multi-mode waveforms (Nolet et al., 1986) can provide a good recovery of the true velocity model of the order of a few percent. By inverting cross-correlograms as secondary observables (Cara & Lévêque, 1987), around $\pm 8\%$ variations can be recovered for Rayleigh waves. In either case, the tomographic models obtained from these waveform inversions seem to be very close or over such limits of velocity perturbation.

1.5 The scope of this thesis

The main emphasis of this thesis is placed on the development of a new approach for surface wave tomography; a *three-stage inversion*, which compensates for the defects of the traditional approach in both regional and global surface wave tomography. Several new techniques, which enable us to exploit the *three-stage inversion*, are fully explained preceding the development of the new approach to tomographic inversion.

In chapter 2, the general features of surface waves and normal modes, which are the basis of surface wave synthesis, are discussed. Although normal modes can be considered as standing waves or free oscillations of a spherical Earth, the superpositions of such modes possess the character of travelling waves and, as a result, synthetic surface waves as well as long-period body waves can be obtained by superposing normal modes. We further investigate surface wave propagation in a 3-D structure using synthetic seismograms based on WKBJ theory with a great-circle approximation. The visualisation of surface wave propagations in laterally heterogeneous media. The effects of lateral heterogeneity on surface wave dispersion and on the behaviour of ray paths are also discussed.

In chapter 3, we develop a new approach to measure multi-mode dispersion of surface waves employing fully nonlinear waveform inversion with a Neighbourhood Algorithm (NA) of Sambridge (1999a). We first invert a multi-mode waveform for a path average 1-D model. Thousands of 1-D models are generated using the NA. We find that different 1-D models, which give reasonably good fit to the observations, provide a very similar character of multi-mode dispersion curves. Thus we will consider such 1-D models as a dispersion estimator rather than as a representation of the actual Earth model.

In chapter 4, a technique of *Fresnel-area ray tracing* is developed by using paraxial ray theory for surface waves in combination with the kinematic and dynamic ray tracing. This method enables us to evaluate the Fresnel zone in 3-D structures efficiently. From

the careful analysis of the nature of stationary-phase fields that are equivalent to Fresnel areas calculated by using the Fresnel-area ray tracing, the approximate influence zone that significantly affects surface wave fields is investigated.

By using the new techniques explained in chapters 3 and 4, we propose a new approach for surface wave tomography in chapter 5, reformulating a method of tomography in three-stage processes. This three-stage inversion is very effective in its computation and has a number of advantaged over the conventional techniques, for example, we can treat *finite-frequency effects, off-great-circle propagation* and *multi-mode dispersion* in a common framework for both regional and global surface wave tomography.

In chapter 6, the three-stage inversion for surface waves is applied to the Australian region. Practical formulations for the new approach are explained and new 3-D Australian models are displayed. With the inclusion of the effects of off-great-circle propagation and the influence zone, the configuration of the velocity structure is improved in the new models. We can also achieve more uniform horizontal resolution since the finite-width around ray paths has been considered.

In chapter 7, we discuss a preliminary work on 2-D sensitivity kernels for surface wave phase speed structures using the Born and Rytov approximations, which will provide a more sophisticated technique for surface wave inversion with considerable increase in computation time. Such an approach should be of significant importance for the further development of the techniques of surface wave tomography.

Finally chapter 8 provides a summary of the work in the thesis and introduces a number of important problems for the future development of tomographic methods using surface waves.

Surface waves in three-dimensional structures

2.1 Introduction

Once an earthquake occurs, the Earth vibrates like a bell with a set of particular frequencies. This phenomenon is well represented by the Earth's "normal modes" (see e.g., Aki & Richards, 1980, chapter 8; Dziewonski & Woodhouse, 1983; Woodhouse, 1996; Dahlen & Tromp, 1998). The concept of such modes is of importance to understand the nature of surface waves in a spherical Earth because surface wave seismograms (and long-period body waves as well) can be synthesised by superposing these normal modes. If we consider small regions in which the curvature of the Earth can be neglected, we can assume a flat Earth rather than a spherical Earth and "surface wave modes" may be used instead of normal modes (e.g., Aki & Richards, 1980, chapter 7; Kennett, 1983, chapter 11; Kennett, 1998b).

In this chapter, we will first briefly summarise features of the Earth's normal modes and synthetic seismograms calculated from normal mode summation. Subsequently, the WKBJ approximation for surface waves based on ray and mode theory is reviewed, and surface wavefields in a 3-D Earth model calculated from the WKBJ theory are displayed. With the approximation that the propagation of long-period surface waves does not deviate significantly from great-circles, the computational effort for calculating surface waves all over the globe can be significantly reduced. We will also show the nature of surface wave dispersion by calculating approximate arrival time of surface wave trains in global group speed maps. The behaviour of surface wave rays in phase speed maps are also discussed. These simple methods for representing surface waves in 3-D structures provide us with a variety of insights into the nature of the surface wavefield in a 3-D laterally heterogeneous medium which cannot readily be obtained in other wavs.



Fig. 2.1. Oscillations of a string with fixed ends.

2.2 General features of normal modes

The concept of modes of oscillations can be readily understood by considering a string (length l) fixed at both ends, like a guitar string (Fig 2.1). When one plucks the string, it vibrates with a particular wavelength (l, l/2, l/3, ...). The motion with the lowest frequency or longest wavelength is called the fundamental mode, and that of higher frequency with one node is the first higher mode, and with two nodes is the second higher mode. The actual tone produced by a stringed instrument is determined by combinations of these modes.

In case of a spherical Earth, the length of the string may be considered as corresponding to the Earth's radius, and one end is fixed at the center of the Earth with the displacement fixed at 0, whilst the other end is the free surface. Unlike a string, the Earth has finite scale in 3-D and the 1-D structures along the radius are considerably heterogeneous, therefore the oscillations of the Earth are much more complicated than the case of a string. There are a large number of modes of the Earth's oscillation as seen in Fig 2.2 and they can be calculated numerically for a spherically symmetric Earth by solving coupled differential equations which are derived from the equations of motion (Takeuchi & Saito, 1972; Aki & Richards, 1980; Woodhouse, 1988; Dahlen & Tromp, 1998). For an isotropic model, such normal modes consist of two types of modes; spheroidal and toroidal modes, which correspond to Rayleigh and Love waves, respectively. All normal modes with frequencies lower than 30 mHz calculated for PREM (Preliminary Reference Earth Model; Dziewonski & Anderson, 1981) are shown in the dispersion diagrams (Fig 2.2). Each point in the diagrams displays a normal mode of the Earth. By taking a summation of all these modes



Spheroidal Mode

Fig. 2.2. Dispersion diagrams for spheroidal modes (top) and toroidal modes (bottom). Vertical axis shows eigenfrequency and horizontal axis shows angular order. Each point corresponds to a normal mode calculated from PREM.



Fig. 2.3. Eigenfunctions ${}_{n}U_{l}$ (solid line) and ${}_{n}V_{l}$ (dotted line) of spheroidal mode (top) and ${}_{n}W_{l}$ of toroidal mode (bottom) with varying overtone number n. Values above each eigenfunction show the eigen period for the mode. Angular order l is fixed at l = 30 which corresponds to wavelength of about 1300 km.

with suitable excitation factors, we can construct synthetic seismograms including body and surface waves for particular frequency ranges as described in section 2.3.

To represent the normal modes, we generally use the notation ${}_{n}S_{l}$ for spheroidal modes and ${}_{n}T_{l}$ for toroidal modes, where n (= 0, 1, 2, ...) is the overtone (or higher mode) number and l (= 1, 2, ...) is angular order. As shown in section 2.3, the angular order l is related to the wavelength λ by $\lambda = 2\pi R/(l + \frac{1}{2})$, where R is the radius of the Earth. Several examples of the eigenfunctions of normal modes computed for PREM are shown in Fig 2.3 and Fig 2.4.

Fig 2.3 shows the variation of the eigenfunctions ${}_{n}U_{l}$ (vertical component) and ${}_{n}V_{l}$ (radial component) of the spheroidal modes and ${}_{n}W_{l}$ (transverse component) of the toroidal modes with varying overtone number n and Fig 2.4 shows that with varying angular order l. Such eigenfunctions can be considered as indicating energy profiles of the modes excited by an earthquake. As the overtone number n increases (Fig 2.3), the number of oscillation of the eigenfunctions increases and the energy reaches the deeper parts of the Earth. This is the reason why higher-mode surface waves can resolve deeper parts of the mantle than the fundamental mode. A peculiar mode (e.g., ${}_{3}S_{30}$ in Fig 2.3), which has prominent



Fig. 2.4. Same as Fig 2.3 but with varying angular order l. Overtone number n is fixed at l = 2 which corresponds to the second higher modes.

amplitude at a particular discontinuity, is a Stoneley mode and represent a trapped mode at solid-liquid boundaries such as the core-mantle boundary (CMB) and the inner-core boundary (ICB). When the overtone number n is fixed (Fig 2.4), we can see the variation of the eigenfunctions with varying period. As the period becomes longer, the shape of eigenfunctions is elongated toward the deeper part of the Earth, which suggests that the oscillations with longer period are sensitive to deeper structures.

In the presence of lateral heterogeneity, we need to consider three dimensional variations of the Earth structure. When the seismic parameters, such as density ρ , P-wave speed α and S-wave speed β , are perturbed, the eigenfrequencies ω of the Earth's normal modes are also perturbed, and this perturbation can be represented to first order as,

$$\frac{\delta\omega}{\omega} = \int_0^R \left\{ K_\rho(r) \frac{\delta\rho}{\rho} + K_\alpha(r) \frac{\delta\alpha}{\alpha} + K_\beta(r) \frac{\delta\beta}{\beta} \right\} dr,$$
(2.1)

where $\delta\rho$, $\delta\alpha$ and $\delta\beta$ are the perturbations of density, P-wave speed and S-wave speed, respectively, $\delta\omega$ is perturbation of the eigenfrequency and R is the radius of the Earth. The integration is taken from the centre of the Earth to the surface. K_{ρ} , K_{α} and K_{β} are sensitivity kernels which work as weight functions applied to the perturbations of parameters. Several examples of sensitivity kernels which correspond to the normal modes



Fig. 2.5. Sensitivity kernels K_{ρ} (dotted line), K_{α} (dashed line) and K_{β} (solid line) for normal modes in Fig 2.3.

in Fig 2.3 are shown in Fig 2.5. The sensitivity kernels for each mode can be calculated from the corresponding eigenfunctions and explicit formulations are given in, e.g., Takeuchi & Saito (1972), Woodhouse (1976) and Dahlen & Tromp (1998). As seen in Fig 2.5, K_{α} does not appear for the toroidal mode, since toroidal oscillations do not depend on P-wave speed, whilst K_{β} is dominant in every case, suggesting that surface-wave propagation depends mainly on S-wave speed even for spheroidal modes. Using (2.1), we can compute the phase speed perturbation δc for a mode as follows,

$$\frac{\delta c}{c} = \frac{c}{C} \frac{\delta \omega}{\omega},\tag{2.2}$$

where $C = d\omega/dk$ is the group speed and k is the wavenumber for the mode. When we investigate the surface wave propagation in laterally heterogeneous media, it is convenient to extract phase speed structure from a 3-D model since the ray paths of surface waves depend on such phase speeds. Phase speed maps are apparently 2-D because they do not depend on the depth explicitly but on frequency. However, they include 3-D information via the sensitivity kernels in (2.1).

2.3 Synthetic seismograms by modal summation

The normal modes discussed so far are basically "standing waves" with stationary phases which correspond to free oscillations of the spherical Earth. Now let us examine how the superposition of such standing waves form "travelling waves", particularly for the surface wave case.

The displacement \mathbf{u} at the Earth's surface can be derived by the superposition of normal modes in a spherically symmetric Earth (e.g., Aki & Richards, 1980, Dziewonski & Woodhouse, 1983) from

$$\mathbf{u}(\mathbf{x},t) = \sum_{k} A_k(t) \ \mathbf{s}_k(\mathbf{x}),\tag{2.3}$$

where $\mathbf{s}_k(\mathbf{x})$ is the *k*th normal mode at receiver \mathbf{x} . The amplitude for the mode is given by,

$$A_k(t) = M_{ij} e_{ij}^{(k)^*}(\mathbf{x}_s) \left[\frac{1 - \exp\left(\omega_k t/2Q_k\right) \cos\omega_k t}{\omega_k^2} \right],$$
(2.4)

where ω_k is an eigenfrequency and Q_k is a quality factor which defines the decay rate of the *k*th mode, M_{ij} is a moment tensor of a source and $e_{ij}^{(k)^*}$ is complex conjugate of strain tensor at the source location \mathbf{x}_s .

The displacement field of normal modes \mathbf{s}_k in a spherical polar coordinate (r, θ, ϕ) can be expanded in spherical harmonics Y_l^m with angular order l and azimuthal order m as follows,

$$\mathbf{s}_k = \hat{\mathbf{r}} \ U^k + \nabla_1 \ V^k - (\hat{\mathbf{r}} \times \nabla_1) \ W^k, \tag{2.5}$$

where $\nabla_1 = \hat{\theta} \partial_{\theta} + \hat{\phi}(\sin \theta)^{-1} \partial_{\phi}$ and the scalar functions U^k , V^k and W^k are represented by,

$$U^{k} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} {}_{k} U_{l}(r) Y_{l}^{m}(\theta, \phi), \qquad (2.6)$$

$$V^{k} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} {}_{k} V_{l}(r) Y_{l}^{m}(\theta, \phi), \qquad (2.7)$$

$$W^{k} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} {}_{k}W_{l}(r) Y_{l}^{m}(\theta, \phi).$$
(2.8)

It should be noted that the eigenfunctions ${}_{k}U_{l}$, ${}_{k}V_{l}$, ${}_{k}W_{l}$ do not depend on azimuthal order *m* because of "degeneracy" (see e.g., Aki & Richards, 1980) which arises as a result of the symmetry of a spherical Earth. The fully normalised spherical harmonics Y_{l}^{m} take the form,

$$Y_l^m(\theta,\phi) = (-1)^m \left[\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}\right]^{1/2} P_l^m(\cos\theta) e^{im\phi},$$
(2.9)

where $P_l^m(\cos\theta)$ are associated Legendre functions.

The quantitative relation between normal modes and travelling waves can be now obtained by using the asymptotic character of the spherical harmonics for large angular order l. Let us consider a simple case of spheroidal modes generated by a point source which is located at the pole $\theta = 0$ (Aki & Richards, 1980; Woodhouse, 1996). In this case, the rcomponent of displacement becomes,

$$u_{r}(r,\theta,\phi,t) = \sum_{n} A_{n}(t) \sum_{l} \sum_{m} {}_{n}U_{l}(r) Y_{l}^{m}(\theta,\phi).$$
(2.10)

Note that the *l* dependence in A_n via strain components of (2.4) (see e.g, Dziewonski & Woodhouse, 1983) is omitted since it does not affect the travelling wave representation. For fixed *m* and large *l*, the asymptotic expansion of $Y_l^m(\theta, \phi)$ is given by (e.g., Woodhouse 1996),

$$Y_l^m(\theta,\phi) \sim \frac{1}{\pi} (\sin\theta)^{1/2} \cos\left[\left(l + \frac{1}{2}\right)\theta + \frac{1}{2}m\pi - \frac{1}{4}\pi\right] e^{im\phi}.$$
 (2.11)

Since the source is assumed to be at the pole, θ corresponds to the angular distance from source to receiver, so that the epicentral distance is $\Delta = R \ \theta$. We can therefore identify the horizontal wavenumber k as

$$k = \left(l + \frac{1}{2}\right)/R.$$
(2.12)

From (2.11) and (2.12), the displacement (2.10) derived from modal summation can be considered as travelling surface wave which propagate with phase speed c,

$$c = \frac{\omega}{k} = \frac{\omega R}{(l+1/2)}.$$
(2.13)

An example of observed and synthetic waveforms for the vertical components of longperiod Rayleigh waves calculated from modal summation are shown in Fig 2.6. The synthetic seismograms are calculated for the PREM model. The excitation of the modes depends greatly on depth of the source. Generally, deep events excite higher modes quite well, whereas fundamental modes are dominant for shallow events. In Fig 2.6, the fundamental and the first few higher modes are well excited.

2.4 Synthetic seismograms with WKBJ approximation

An alternative way to calculate synthetic seismograms along a ray can be achieved by employing the WKBJ approximation (e.g., Tromp & Dahlen, 1992a,b). We briefly summarise the formulation for surface waves based on the WKBJ approximation in frequency domain,



Fig. 2.6. Vertical component of observed seismogram at NWAO (top) for the event at Vanuatu islands (lat=14.57°S, lon=167.19°E, depth=171.4 km), and full synthetic seismogram (second row) computed for PREM. All the seismograms are band-pass filtered within 8-30 mHz, and waveforms for the first ten modes are shown separately.

following the description of Dahlen & Tromp (1998). In the WKBJ approximation, the surface displacements are assumed to be represented as

$$\mathbf{u}(\mathbf{x},\omega) = \sum_{mode \ orbit} A(\omega) \exp(-i\psi(\omega)). \tag{2.14}$$

The amplitude term A and the phase term ψ can be divided into source, path and receiver terms,

$$A = A_r A_p A_s, (2.15)$$

$$\psi = \psi_r + \psi_p + \psi_s, \tag{2.16}$$

where suffices r, p and s represent receiver, path and source. Each term is represented as follows:

a) Source term

$$A_s \exp(-i\psi_s) = -\frac{i}{\omega} (\mathbf{M} : \mathbf{E}_s^*) \exp(-i\frac{\pi}{4}).$$
(2.17)

For Love waves, the contraction of the moment tensor \mathbf{M} and the conjugate source strain tensor \mathbf{E}_s^* is represented as

$$\mathbf{M} : \mathbf{E}_{s}^{*} = i(\partial_{r}W_{s} - r_{s}^{-1}W_{s}) \left(M_{r\theta}\sin\zeta_{s} - M_{r\phi}\cos\zeta_{s}\right) -r_{s}^{-1}k_{s}W_{s}\left[\frac{1}{2}(M_{\theta\theta} - M_{\phi\phi})\sin 2\zeta_{s} - M_{\theta\phi}\cos 2\zeta_{s}\right],$$
(2.18)

and for Rayleigh waves,

$$\mathbf{M} : \mathbf{E}_{s}^{*} = M_{rr}\partial_{r}U_{s} + r_{s}^{-1}(U_{s} - \frac{1}{2}k_{s}V_{s}) (M_{\theta\theta} + M_{\phi\phi}) + i(\partial_{r}V_{s} - r_{s}^{-1}V_{s} + r_{s}^{-1}k_{s}U_{s})(M_{r\phi}\sin\zeta_{s} + M_{r\theta}\cos\zeta_{s}) - r_{s}^{-1}k_{s}V_{s} \left[M_{\theta\phi}\sin2\zeta_{s} + \frac{1}{2}(M_{\theta\theta} - M_{\phi\phi})\cos2\zeta_{s} \right],$$
(2.19)

where ζ_s is the azimuth measured counter-clockwise from south, r_s is the radius of the source depth, k_s is the wavenumber at the source, and U_s, V_s and W_s are eigen functions at the source. The notation used for the moment tensor is that for Harvard CMT solutions. b) Path term:

$$A_p \exp(-i\psi_p) = (8\pi k |\sin\Delta|)^{-1/2} \exp(-\int_0^\Delta \frac{\omega}{2CQ} d\Delta) \exp(-\int_0^\Delta k d\Delta + M\frac{\pi}{2}), \qquad (2.20)$$

where k, C and Q are the local wavenumber, group speed and quality factor, which are computed for local 1-D structures. Δ is the epicentral distance, M = s - 1 and s is the ray orbit number (s = 1, 2, 3, ...). On a spherical Earth, the epicentral angular distances for odd and even orbit are given by

$$\Delta = \begin{cases} (s-1)\pi + \phi &: \text{ odd} \\ s\pi - \phi &: \text{ even,} \end{cases}$$
(2.21)

where ϕ is the angular distance of a minor arc. The integration term over local wavenumber in (2.20) can be represented as

$$k_{sum} = \int_0^\Delta k d\Delta = \begin{cases} \frac{1}{2}(s-1)k_{gc} + k_{minor} &: \text{ odd} \\ \frac{1}{2}sk_{gc} - k_{minor} &: \text{ even,} \end{cases}$$
(2.22)

where

$$k_{gc} = \oint k d\phi, \quad k_{minor} = \int_0^\phi k d\phi. \tag{2.23}$$

The other integration in terms of attenuation in (2.20) can be expressed in a similar way. c) Receiver term:

$$A_r \exp(-i\psi_r) = \begin{cases} U &: vertical \\ -iV &: radial \\ iW &: transverse, \end{cases}$$
(2.24)

where U, V and W are the eigenfunctions at receiver.

The normalisation convention of eigenfunctions in the above formulation is as follows,

$$cCI_1 = 1 \tag{2.25}$$

where c is phase speed, C is group speed and

$$I_{1} = \begin{cases} \int_{0}^{a} \rho(U^{2} + V^{2})r^{2}dr & : Rayleigh wave, \\ \int_{0}^{a} \rho W^{2}r^{2}dr & : Love wave. \end{cases}$$
(2.26)

2.5 Surface wave propagation in a 3-D model

For a complete representation of surface wavefields in a 3-D structure, some numerical scheme such as finite difference and spectral methods have been the most popular approaches. However, such numerical approaches require a huge amount of computation. As a simplified way to obtain surface wavefields in a global 3-D structure, a ray theoretical approach may be useful since it does not take too much computation time, although we cannot consider scattering effects from heterogeneous structure.

In order to visualise the nature of surface wave propagation in a 3-D structure, we have adopted a ray theoretical approach to calculate surface wavefields on a sphere. A global 3-D Earth model, 3SMAC (Nataf & Ricard, 1996; Ricard et al., 1996), is used for the surface wave computation. Some information, which is not contained in the 3SMAC model (e.g., lower mantle structure, a quality factor Q_{κ} for compressional waves), is substituted by PREM. The phase speed maps of Love and Rayleigh waves at 100 seconds computed from 3SMAC are shown in Fig 2.7.

The WKBJ response is calculated for 16200 points on the sphere, with 2° sampling in



Fig. 2.7. Phase speed maps of Love (top) and Rayleigh (bottom) waves at 100 seconds calculated from 3SMAC by using anisotropic PREM as a reference model. Note that pale blue regions in the Love wave model show slight low velocity anomaly.

both azimuth and distance from the source. Note that the total number of grids is the same as 3SMAC, but the location of the grid points are different. This can be interpreted as that the source is assumed to lie at the North Pole and surface displacements are calculated at crossing points of meridians and parallels of latitude at every 2 degree. The synthetic seismograms are then band-pass filtered within the range 6 - 15 mHz.

A source is introduced near Halmahera, Indonesia with the source mechanism shown in Table 2.1, and the radiation pattern of Love and Rayleigh waves calculated from the absolute values of (2.17) are shown in Fig 2.8.

Snapshots of fundamental mode Rayleigh and Love waves at 600, 800 and 1000 seconds

event location	lat: 1.2°	lon: 127.8°	depth: 15 km $$
moment tensor $(exp=26)$	$M_{rr} = -0.34$ $M_{r\theta} = -0.04$	$M_{\theta\theta} = 0.97$ $M_{r\phi} = -1.30$	$M_{\phi\phi} = -0.63$ $M_{\theta\phi} = -2.85$

Table 2.1. Source parameters for the event used in this study.



Fig. 2.8. Radiation patterns of Love (left) and Rayleigh waves (right) calculated from the absolute value of the source term in (2.17).

are displayed in Fig 2.9. The black and white stripes are clearly distorted in and around Australia compared to the surrounding oceanic regions. Although the evolution of the surface-wave fronts may not be clear, the dispersive character of surface waves clearly appear as thick circular belts. Unfortunately, because of the different radiation patterns for Love and Rayleigh waves, they do not sample the same regions with the same sensitivity. Both types of surface waves, however, are strongly affected by the lateral heterogeneities of the Australian Continent. For example, a phase advance in the western part of Australia compared to the eastern part is apparent for Love waves, and strong effects from the continent-ocean boundary are clearly seen at the western margin of the Australian Continent for Rayleigh waves.

Fig 2.10 shows snapshots of Love and Rayleigh waves at 2500, 3000 and 3500 seconds. Note that the view point is different from Fig 2.9, but the surface waves are radiated from the same event. The distortion of wavefronts due to lateral heterogeneities are apparent for both Love and Rayleigh waves. The Love waves are affected by the low phase speed in the north Atlantic and high phase speed in Greenland and North America. Rayleigh waves radiated from this event propagate very interesting paths. By an elapsed time of 2500 seconds, Rayleigh wave near the Black Sea and the Caspian Sea shows a slight phase



Fig. 2.9. Phase speed maps of Love (top left) and Rayleigh (top right) waves at period of 100 seconds. Snapshots of fundamental mode Love (middle) and Rayleigh (bottom) waves passing through the Australian Continent. The source location is marked with the cross.







Fig. 2.10. Same as Fig 2.9 but passing through the Eurasian and African Continents. Note that the event is the same as Fig 2.9.



Fig. 2.11. Group speed maps of Larson & Ekström (2001) for fundamental-mode Love (left panels) and Rayleigh (right panels) at 50 s (top) and 100 s (bottom).

delay caused by passing through slow velocity anomaly regions around the Himalayas. This delayed portion is further affected by the slow velocity anomaly around the Mediterranean, whereas the surrounding parts of the wavefront pass through the high velocity region of Europe and African Continent, resulting in a strong deviation of the wavefront as seen in the snapshot at 3500 seconds.

2.6 Surface wave dispersion in 3-D media

In order to further investigate the dispersive behaviour of surface waves in laterally heterogeneous structure, we calculate the wavefronts for surface waves by estimating surface-wave travel times using global group speed models of Larson & Ekström (2001) in Fig 2.11.



Fig. 2.12. Representation of fundamental-mode Love and Rayleigh waves radiated from a source near coast of the southern Peru, which is indicated with a cross.

Approximate arrival time of surface wave trains are estimated from these group speed maps at periods ranges between 35 and 100 seconds. Surface wave arrivals that are equivalent to wavefronts are displayed in Fig 2.12 to 2.14 with varying width of line segments, which is wider for longer periods. We simply evaluate the approximate arrival times from the group speed maps without considering the effects of radiation from the source.


Fig. 2.13. Same as Fig 2.12 but at 30 minutes with different view points.

It is apparent that the behaviour of surface wave dispersion depends strongly on the paths. Since a source located near the coast line of the southern Peru corresponding to the very large event in June 2001 is chosen, we can clearly see the significant differences in dispersion characters of surface waves radiated in different directions. We can see more severe dispersion for both Love and Rayleigh waves radiated to the east of the source, propagating inside the South American Continent, whilst surface waves radiated towards the west, travelling in the Pacific Ocean, do not show such strong dispersion.

We can also see in Fig 2.12 that the nature of surface wave dispersion is different for wave paths passing through the ocean and continent. For the paths in the Pacific Ocean, shorter period waves (thiner lines) travel faster than the longer period waves (thicker lines), whereas, for the paths in the South American Continent, longer period waves tend to travel ahead of shorter period waves. Such phenomena are strongly affected by the differences in crustal and uppermost mantle structures.

In Fig 2.13, surface waves at the same elapsed time of 30 minutes are displayed with different view angles. The differences in the dispersive natures explained above are clearly identified. Moreover, we can also see different character in the Love and Rayleigh waves. Rayleigh waves at 30 minutes show a similar length of ray segments for paths radiated toward both the east and west from the source, indicating the existence of strong dispersion for both paths, although the behaviour of long and short period waves are opposite. However, Love waves show significant differences in dispersion depending on the paths in



Fig. 2.14. Same as Fig 2.12 but at 66 minutes looking at the map from the other side of the source.

Fig 2.13, that is, we cannot see significant dispersion in the paths in the Pacific, whereas there is much more apparent dispersion in the paths passing through the other regions.

In Fig 2.14, wavefronts at 66 minutes are displayed, showing surface waves coming into an antipode on the other hemisphere of the Earth. We can see obvious distortion of wavefronts, indicating more significant effects from the lateral heterogeneity on paths travelling long distances.

Although we have employed simple methods to estimate the surface wavefields in the previous section and the the nature of dispersion in this section as a means of visually assessing the nature of surface waves, the results provide interesting insights into the surface wave propagation in the laterally heterogeneous Earth.

2.7 Surface wave ray tracing in phase speed maps

One of the important effects of lateral heterogeneity on surface wave propagation is that ray paths depart from the great-circle between the source and receiver. This is particularly important in applications of the geometrical ray theory to surface wave tomography, in which the ray paths have been generally supposed to be the great-circle. In this section, we investigate the behaviour of surface wave ray paths in the Australian region where there are significant horizontal variations in shear wave speed due to its unique structural setting.

Because of the dispersive nature of surface waves, ray paths depend on the frequency of



Fig. 2.15. Ray paths traced on Rayleigh-wave phase speed maps for fundamental (top), first-higher (middle) and second-higher (bottom) modes together with corresponding sensitivity kernels for P velocity (K_{α}) , S velocity (K_{β}) and density (K_{ρ}) at 100 seconds. Rays are radiated from a source at the center of New Guinea Island (4.5°S, 143.5°E) with varying azimuth from 90° to 270° for every 3°. Reference phase speeds are 4.09 km/s for the fundamental mode, 5.87 km/s for the first higher mode and 7.15 km/s for the second higher mode.

interest. Therefore the ray trajectories for surface waves are mainly controlled by phase speed distributions as a function of frequency (Woodhouse, 1974), which can be derived from a 3-D velocity structure. In order to estimate two-dimensional phase speed structure, we employ an 3-D SV velocity model of the Australian continent of Debayle & Kennett (2000a) as an upper-mantle shear wave speed structure and the rest of the structural parameters (P wave speed, density and Q) are derived from PREM. The crustal corrections are made using 3SMAC (Nataf & Ricard, 1996) and PREM is used as a reference model to compute phase speed perturbations.

Surface wave rays are traced in the Rayleigh-wave phase speed maps for the first three modes (the fundamental, first-higher and second-higher modes) at 100 and 40 seconds period using phase speeds estimated from the 3-D shear wave speed model. In Fig 2.15 and 2.16, the surface wave rays traced in these phase speed structures are drawn on each phase speed map together with the corresponding sensitivity kernels of the dispersion with respect to P and S wave speed and density. In these maps, we can see the clear effect of lateral heterogeneities upon the behaviour of ray paths traversing the phase speed maps, and this effect becomes more significant in phase speed models that are sensitive to the shallow part of the mantle.

At 100 seconds period in Fig 2.15, the deviations of rays from the great circle are not so large except for a few regions such as the ocean-continent boundaries and in eastern Australia where strong velocity jumps exist. For the higher-modes, which sample much broader ranges of depth in the upper mantle, the perturbations of phase speed are smaller and as a result the path deviations are weaker.

In the shorter period models at 40 seconds period (Fig 2.16), the phase speed perturbations are more pronounced, especially for the fundamental-mode, and off-great-circle propagation and multi-path effects become apparent in this model. For the first-higher mode, path deviations are still obvious, whereas for the second-higher mode, they are not so significant. Thus, we may say that the off-great-circle propagation may need to be taken into account only for the fundamental mode and the first-higher mode in shorter period models, and the higher modes (from the second upwards) can be described quite well by the great-circle approximation.

It is worth noting that the appearance of the fundamental-mode phase speed maps at 40 seconds differs considerably from the higher-mode maps. This is because shorter-period fundamental-mode surface waves are sensitive only to the shallow layers (the crust and uppermost mantle above 150 km), whilst the higher-modes and long-period fundamental modes are sensitive also to much deeper structure, where velocity perturbations are likely



Fig. 2.16. Same as Fig 2.15 but at 40 seconds. Reference phase speeds are 3.93 km/s for the fundamental mode, 4.87 km/s for the first higher mode and 5.39 km/s for the second higher mode.



Fig. 2.17. Same as Fig 2.16 (top) but for sources at 16° S, 124° E (top) and 22.75° S, 171.48° E (bottom).

to be smaller. Therefore, the treatment of short-period surface waves is more difficult than the longer-periods because of the stronger lateral heterogeneity in the shallow layers.

Fig 2.17 shows rays in the fundamental mode Rayleigh-wave phase speed model at 40 seconds with different source locations. These figures shows that distortion of the wavefield depends significantly on the position of the source. In the top panel of Fig 2.17, the source is located near the north-western edge of the Australian continent, and the rays radiated from this source propagate with small distortion inside the continent. However, the rays begin to be bent noticeably offshore especially in the Coral Sea (to the north-east of



Fig. 2.18. Two-point ray shooting for three global stations, CAN (top), CTAO (middle) and NWAO (bottom) with the fundamental-mode Rayleigh wave phase speed map at 40 seconds in Fig 2.16 (top). Green lines show actual ray-paths and black dotted lines the corresponding great-circle.

Australia). The ray paths shot from a source in the east in New Caledonia (Fig 2.17 bottom) are severely bent by the large velocity gradient in the Coral Sea and the northern Tasman Sea, causing conspicuous effects of focusing and defocusing.

The results from two-point ray shooting experiment for three global stations (CAN,

CTAO and NWAO) in a 40 seconds model are displayed in Fig 2.18. In most cases, the deviation from the great-circle is not so large (the arrival-angle deviations are within 5 degrees), but for some paths, such as a ray path arriving at NWAO coming from eastern New Caledonia in the bottom panel of Fig 2.18, the arrival angles are about 10 degrees away from the great-circle and the ray separates from the great-circle by about 200 km at most. These results suggest that the great-circle approximation is not terribly wrong, since the minimum scale-length of the lateral heterogeneity in the regional tomography models is much longer than 200 km, and thus the ray path deviations as seen in these ray-shooting experiment should not alter the tomography models significantly.

We should note that only the shortest path can be obtained by this kind of two-point ray shooting. In other words, even if there are multiple rays arriving at a station, we cannot calculate them simply, although we can expect that the shortest paths should convey the largest surface-wave energy.

The effects of off-great-circle propagation of surface waves are investigated throughout this thesis. In particular, in chapter 4, ray-path bending is treated in association with the studies on the influence zone around surface wave paths, and is further incorporated into tomographic inversions for phase speed maps in chapter 6. Nonlinear waveform inversion for surface waves - Application to multi-mode dispersion measurements

3.1 Introduction

Multi-mode information is essential for enhancing the vertical resolution of surface-wave tomography models. In most regional studies, a common approach is to invert waveforms of multi-mode surface waves for path-specific 1-D models (e.g., Cara & Lévêque, 1987; Nolet, 1990). Although such a method should have a good resolution in depth due to constraints from higher modes, the resultant path-specific 1-D models are quite sensitive to the model parameterisation and to the reference model used to start the nonlinear inversion, resulting in some non-uniqueness of 1-D models which achieve adequate fit to the observations.

Direct measurements of group and phase dispersion have also been utilised in a number of studies (e.g., Ekström, Tromp & Larson, 1997; Ritzwoller & Levshin, 1998) and allow the extraction of stable results from observations without any interference from the style of model parameterisation, although the direct measurements of dispersion can only be readily applied to fundamental modes.

Techniques for measuring higher-mode phase speeds have mainly been based on the concept of "mode separation". One of the traditional ways to isolate overlapping higher modes is to apply a frequency-wavenumber filter to stacked waveforms observed in a seismograph network (e.g., Nolet, 1975; Cara, 1978).

Measuring multi-mode dispersion from a single seismogram is not a straightforward issue. Stutzmann & Montagner (1993) used a set of seismograms recorded at a single station with several sources at different depths in a small epicentral area to obtain reliable multimode dispersion measurements. This requirement of similar source-receiver pairs reduces the number of available paths dramatically. Van Heijst & Woodhouse (1997) developed a "mode-branch stripping" technique by using mode-branch cross-correlation functions. In 36 this method, phase speeds for a mode-branch are measured by fitting the cross-correlation function for the mode, then the contribution to the mode-branch seismogram is removed from the observed seismogram and the process is repeated for the next mode-branch. This technique is quite effective especially in case of longer paths for which higher-mode branches do not overlap each other on the seismogram (van Heijst & Woodhouse, 1999). However, it cannot readily be applied to regional studies for which the most paths are shorter than 30° and individual higher-mode contributions can hardly be distinguished in a seismogram.

In this chapter we propose a new technique of multi-mode dispersion measurement using nonlinear waveform inversion for surface waves, especially for regional surface waves. The process of waveform inversion depends on a knowledge of the source mechanism and is quite sensitive to the starting model with consequent ambiguities in the model. Therefore, there are advantages in adopting a direct nonlinear approach (without linearisation) which does not require the evaluation of derivatives with respect to the model parameters. We adopt the Neighbourhood Algorithm of Sambridge (1999a) (hereafter referred to as NA) that enables us to explore a model parameter space so as to best fit the observations.

We take a different viewpoint from the current styles of waveform inversion, and do not consider the path-specific 1-D models as a direct representation of the Earth model, but instead we interpret them as providing implicit information on multi-mode dispersion for the source-receiver path. If the perturbations from the reference model are weak it may be justified to interpret the 1-D models themselves as an average along the path, but the waveform inversion does not depend on this assumption (Kennett & Yoshizawa, 2002). The 1-D models derived from waveform inversions depend significantly on the parameterisation and the reference model. Even though models differ, synthetic waveforms for the models match well to observations, suggesting that the multi-mode dispersion is well represented through the process of waveform fitting. The multi-mode phase speeds derived for the various paths can be used to reconstruct multi-mode dispersion maps which will provide crucial information for reconstructing 3-D shear wavespeed structure.

3.2 Method of nonlinear waveform inversion

The process of waveform inversion is highly nonlinear and the results depend on a knowledge of source mechanism, the parameterisation of the model and the choice of a reference model. Thus a fully nonlinear approach, which does not require any calculations of derivatives with respect to model parameters, is desirable for the purpose of waveform matching. We adopt the Neighbourhood Algorithm (NA) of Sambridge (1999a) as a global optimiser which explores the model parameter space to find models with a good fit to the data. The procedure of nonlinear waveform fitting using the NA can be summarised as follows,

- (1) Generate a path-specific 1-D shear wavespeed model using NA
- (2) Compute a synthetic seismogram for the 1-D model
- (3) Calculate misfit between synthetic and observed waveforms
- (4) Repeat (1) to (3) until N models are calculated
- (5) Compute phase dispersion from the best-fit 1-D model
- (6) Estimate reliability of the dispersion measurement

3.2.1 Neighbourhood Algorithm

The NA is based on simple principles and can be used as global optimiser. The details of the NA are fully described in Sambridge (1999a), and here we only briefly explain the method. The feature of the NA is that model parameter space is divided into Voronoi (nearest neighbour) cells defined by a suitable distance norm (usually L_2) and a search is made over models within these cells with the aim of finding smaller misfit. At each stage (iteration) Voronoi cells are uniquely defined by the previous samples. These irregular polyhedra guide subsequent samples and the algorithm is able to concentrate sampling in favourable regions of parameter space. Only two tunable parameters are necessary, and no derivatives with respect to the model parameters are required. The Neighbourhood Algorithm takes the following form:

- (a) Generate a set of n_s models uniformly in parameter space.
- (b) Calculate misfit for the latest n_s models and choose n_r models with smaller misfit of all generated models.
- (c) n_s new models are generated from a random walk in the Voronoi cell of each of n_r chosen models.
- (d) Repeat (b) and (c).

As mentioned in Sambridge (1999a), the choice of NA parameters n_s and n_r is arbitrary and there is as yet no quantitative ways to determine the optimal values of these parameters. We have performed a number of trials with various combinations of n_s and n_r , and we decided to use the NA method with $n_s = 10$ and $n_r = 5$. 300 iterations are performed so that 3000 models are generated for each inversion.

3.2.2 Waveform inversion for 1-D models

The surface waveforms for a reference model with a high-frequency approximation are approximated as a sum of several modes with simplification of a phase term (e.g., the WKBJ seismogram in chapter 2),

$$u^{0}(\Delta,\omega) = \sum_{j=0}^{J} \mathcal{R}_{j}^{0}(\omega) \exp\left[ik_{j}^{0}(\omega)\Delta\right] \mathcal{S}_{j}^{0}(\omega), \qquad (3.1)$$

where \mathcal{R}_j^0 represents the receiver term, geometrical spreading and attenuation and \mathcal{S}_j^0 source excitation for *j*-th mode branch, k_j^0 is a path-averaged wavenumber along a greatcircle, Δ is epicentral distance and ω is angular frequency.

In the presence of slight lateral heterogeneity, the waveforms can be described approximately by a perturbation of the wavenumber,

$$u(\Delta,\omega) = \sum_{j=0}^{J} \mathcal{R}_{j}(\omega) \exp\left[i\left\{k_{j}^{0}(\omega) + \delta k_{j}(\omega)\right\}\Delta\right] \mathcal{S}_{j}(\omega), \qquad (3.2)$$

where \mathcal{R}_j and \mathcal{S}_j are the receiver and source terms for the actual Earth and $\delta k_j(\omega)$ is the perturbation of wavenumber induced by the variations along the path. From the asymptotic results of Woodhouse (1974) for a smoothly varying model, δk_j is built from the path-average of the local wavenumber perturbations. The final form is equivalent to a perturbation of a 1-D model. Throughout this study, \mathcal{R}_j and \mathcal{R}_j^0 as well as \mathcal{S}_j and \mathcal{S}_j^0 are supposed to be identical.

When we perform fully nonlinear inversions for surface waveforms, it would be desirable to undertake a full recalculation of seismograms by computing normal or surface-wave modes for each new model. However, this places very heavy computational demands and is not a practical way to perform waveform inversions with a global optimisation technique like NA, because such global search methods need to generate a large number of models to find some acceptable models. We have therefore employed perturbation analysis from a reference model, which does not require any recalculation of the normal modes, to update the seismograms for new models derived from the NA.

Assuming that the perturbation of wavenumber depends mainly on shear wavespeed and the perturbation of wavespeed is not so large, δk_j can be represented as the result of the path-averaged shear wavespeed perturbation $\delta\beta(z)$ as a function of depth z,

$$\delta k_j(\omega) = \int_0^a K_\beta^j(\omega, z) \ \delta\beta(z) dz, \tag{3.3}$$

where a is the Earth's radius, $K_{\beta}^{j}(\omega, z)$ is the Frechét derivative or sensitivity kernel of the shear wavespeed for the *j*-th mode (Takeuchi & Saito, 1972; Dahlen & Tromp, 1998) which are calculated for a reference model. In this study, the *P* wavespeed, density and *Q*



Fig. 3.1. Representation of a model-parameter set using B-spline functions. Three discontinuities at Moho, 400 km and 670 km are included in the model parameterisation and, within each layer, shear wavespeed perturbation varies smoothly as a sum of the B-splines.

of a reference Earth model are fixed and no perturbation of these variables are considered, since they have only little influence on the phase speed perturbation for surface waves in the intermediate period range (50-130 s) which is of interest in our study. $\delta\beta$ can be expanded into a set of B-spline functions as,

$$\delta\beta = \sum_{i=M} b_i B_i(z),\tag{3.4}$$

where M is the total number of parameters, $B_i(z)$ is *i*-th B-spline function and the corresponding coefficient b_i is a model parameter which should be obtained from the NA sampling.

An example of the parameterisation of crust and upper mantle using the B-splines is shown in Fig 3.1. We divide the shear wavespeed model into four layers with three boundaries at the depth of Moho, 400 and 670 km where $B_i(z)$ is discontinuous. The number of parameters in each layer is adjustable for each inversion (M_1 : 0 - Moho, M_2 : Moho - 400 km, M_3 : 400 to 670 km, and M_4 : below 670 km, the total number of parameters $M = \sum_{i=1}^{4} M_i$).

One of the important factors in waveform inversion based on the perturbation theory is to choose an appropriate reference model. In this study, we mainly use PREM (Dziewonski & Anderson, 1981) or PREMC whose upper mantle structure is modified to provide a better representation of continental regions. The discontinuity at 220 km depth in the PREM model is modified so that shear wavespeeds around 220 km are smoothly varying. The crustal structures are corrected using the 3SMAC model (Nataf & Ricard, 1996). For regional surface-wave paths passing through regions with strong velocity anomalies, such spherical Earth models cannot be the best choice for the waveform inversion. Since we treat each observation independently, we do not need to use a single or a particular reference model for different paths. We have devised a procedure for obtaining a pathspecific reference model by assessing the phase-speed perturbation of the fundamentalmode surface waves using a spherical Earth model. The details of the technique are described in section 3.2.4.

3.2.3 Fitting multi-band-pass filtered waveforms and envelopes

Unlike traditional linearised inversion, NA does not require any derivatives with respect to model parameters. Therefore any type of misfit function can be used to measure the difference between observed and synthetic seismograms. In order to obtain the best fit waveform for all frequency ranges of interest, several band-pass filters are applied to seismograms and a set of filtered seismograms with different frequency ranges, $(F_iu(t), i =$ $1, n_f$, where F_i is the *i*-th filter and n_f is number of frequency ranges), is generated for both observed and synthetic seismograms (Fig 3.2). Before applying the band-pass filters, a time window is extracted with appropriate group velocity ranges so that several higher modes as well as fundamental mode are included in a time series.

When the difference between a reference model and a true model is significant, "phasecycle skip" can be caused by $2n\pi$ ambiguities in phase and this obscures the fit to the waveforms. This problem is common in direct phase measurement methods, and the problem can be generally cured by organising the measurements of the phase of waveforms from lower to higher frequency. This approach assumes that the phases of the seismograms are smoothly varying with frequency and that the phase perturbation at lower frequencies should not be larger than π . Here, all band-pass-filtered seismograms are inverted at the same time, so that the traditional approach cannot be used. Instead, we introduce the envelopes, $E\{F_iu(t)\}$, for each filtered seismogram, and match these as well. This gives a significant improvement in waveform fitting when higher-modes are included in a time window and several peaks exist in an envelope (Fig 3.2). The introduction of envelope fitting can be regarded as fitting the group slowness, as well as the phase slowness which dictates the details of waveforms themselves.

The misfit functions are defined from the filtered seismograms $F_i u(t)$ and their envelopes $E\{F_i u(t)\}$ as follows,

$$\Phi = \sum_{i=1}^{n_f} \int \left[\left| F_i u^{obs}(t) - F_i u^{syn}(t) \right|^p + w_i \left| E\{F_i u^{obs}(t)\} - E\{F_i u^{syn}(t)\} \right|^p \right] dt,$$
(3.5)

where w_i is the weighting factor for the *i*-th filtered envelope. *p* represents the order of the misfit norm. We adopt an L_3 norm (p = 3) to measure the difference between observed and synthetic waveforms since this is very sensitive to discrepancies in the waveforms or envelopes. We have employed five to seven band-pass filters with overlapping frequency ranges. The envelope fit is helpful for stabilising waveform fitting by avoiding phase cycle



Fig. 3.2. Examples of waveform and envelope fits for six frequency ranges before (left column) and after (right column) NA inversion.

skips, especially when we apply an appropriate set of weighting factors $\{w_i\}$. Too large or too small $\{w_i\}$ tends to counteract the waveform fit. After several trials, we decided to use $w_i = 1.5$ for all the waveform inversions in this study.

Since the ranges of frequency and the relative amplitude of the mode branches contained in a seismogram strongly depend on the excitation at the source, weighting factors can be applied to both the filtered seismograms and envelopes to enhance or to reduce the contribution of some particular frequency ranges in the inversion, although we do not take this into account in the current study.

3.2.4 Data adaptive correction for a reference model

A choice of a reference model to initiate waveform inversion is one of the critical problems for 1-D path inversions, especially where the surface waves have passed through regions with strong local heterogeneities. The non-linear inversion with NA is able to retrieve up to \pm 5 % velocity differences quite well. However, the calculation of the ray theoretical seismogram based on perturbation theory does not allow too large a velocity perturbation. We therefore need to use a proper reference model which is not too far away from the bestfitting model.

In regional studies, the most significant velocity variations which affect intermediate to long period surface waves (50 - 130 s) are generally contained in the uppermost mantle above 200 to 250 km. The fundamental mode surface waves in such an intermediate period range can usually be fitted to the synthetics fairly well by perturbing the shear wavespeed structure of the uppermost part of the mantle.

In order to obtain a reference model which is adaptive to the specific paths, we create several Earth models by perturbing the shear wavespeed in the upper mantle above 250 km from modified PREM or PREMC model. The choice of PREM or PREMC is based on the nature of the regions through which the path passes. Before perturbing the initial reference models, we correct the crustal structure using 3SMAC model (Nataf & Ricard, 1996). The P wavespeed, density and Q of the reference model are fixed, and no perturbation of these variables are considered. We then directly measure the phase speed of the fundamental mode surface waves using a single-station method (see e.g., Nakanishi & Anderson, 1984) by comparing the observations with a synthetic seismogram calculated for a perturbed reference model. We generate several perturbed reference models and calculate the average perturbation of the measured phase speed from the new reference model within a certain frequency bands (7 - 20 mHz). The new model with the minimum average phase-speed perturbation are used as a new reference model for the NA inversion.

An example of the process of searching for a new reference model from PREMC model



Fig. 3.3. An example of the data adaptive search for the reference model. (a) Dispersion curves of the fundamental mode Rayleigh wave. The phase speed directly measured from the observations is plotted with circles and dispersion curves calculated from perturbed reference models are drawn in gray dashed lines. (b) 1-D shear wave speed profile of PREMC model and the new reference model with 3 % faster wavespeed in the upper 250 km. (c) Initial waveform fits for original and updated reference models.

is displayed in Fig 3.3. In this example, we generate 10 trial reference models with shear wavespeed perturbation from -5 to 5 % with 1 % increment. The perturbation of fundamental-mode phase speed measured from a 3 % perturbed PREMC model has the minimum average perturbation of 0.29 %. Hence we use the 3 % faster shear wavespeed model as a new reference model for the NA inversion and recalculate the normal modes and their eigenfunctions for this model.

The comparison of initial misfits for different reference models are shown in Fig 3.3(c). The initial misfit for the PREMC model is quite large and major discrepancies are seen for the fundamental mode. However, the synthetic seismograms for the 3 % faster reference model matches well to the observed seismogram even before the NA inversion. This process may seem somewhat coarse, but we do not require rigorous calculations of the

reference model at this stage, because finding a data adaptive reference model is just a preliminary process for the subsequent nonlinear waveform inversion. The objective of the data adaptive procedure for estimating a reference model is to keep the required perturbations within $\pm 2\%$, for which the first-order perturbation theory should work well. The data adaptive selection of a reference model has been adopted in the actual waveform inversion in section 3.5.

3.3 Multi-mode dispersion measurement

3.3.1 Phase speed estimation from 1-D shear wavespeed models

The 1-D models derived from the nonlinear waveform inversion are quite non-unique and several different models can provide reasonably good waveform fit. In this study, we do not interpret such 1-D models obtained from the inversion as an actual Earth model, but we use them as a representation of multi-mode dispersion of surface waves.

The path-averaged phase speed perturbation for *j*-th mode, $\delta c_j(\omega)$, can be simply calculated from the wavenumber perturbation as follows,

$$\frac{\delta c_j(\omega)}{c_j^0(\omega)} = -\frac{\delta k_j(\omega)}{k_j^0(\omega)},\tag{3.6}$$

where $c_i^0(\omega)$ is phase speed for a reference model.

Using the ensemble of models sampled by NA, we can estimate approximate errors in dispersion measurement. Resampling of models using NA would be an appropriate way to estimate errors from the ensemble of models (Sambridge, 1999b), but the resampling process is computationally demanding and like any Bayesian approach we would also need to characterise statistics of all noise processes involved (this is not a trivial task). In this study, we roughly estimate errors of the dispersion measurements from the standard deviations of dispersion curves for the best 1000 models without resampling of the models. Averages of the best 1000 models are very close to the best-fit models, so that this error estimation provides quite reasonable error bars around the estimated phase speeds. However, such rough estimates of errors are not quite sufficient for a quantitative evaluation of the measured phase speeds. Therefore, we introduce the reliability of the measurement as a way to evaluate our dispersion estimates.

3.3.2 Reliability of measured phase speed

One of the problems in dispersion measurement from a single seismogram is that it is not simple to evaluate meaningful errors or the reliability of the measurements. Van Heijst & Woodhouse (1997) proposed a way to estimate the reliability by working with mode-branch seismograms. Following their concept, we now re-define the reliability of the measured phase speeds which are recalculated from the 1-D models obtained through NA inversion.

First let us introduce four types of seismograms; the observed seismogram $u^{obs}(t)$, the full synthetic seismogram $u^{syn}(t)$, the *j*-th mode-branch seismogram $u_j^{syn}(t)$ and a residual seismogram for *j*-th mode $\bar{u}_j^{syn}(t) (= u^{syn}(t) - u_j^{syn}(t))$. Then we further define the corresponding spectrograms; $S^{obs}(\omega, t)$, $S^{syn}(\omega, t)$, $S_j^{syn}(\omega, t)$ and $\bar{S}_j^{syn}(\omega, t)$ in the frequencytime (F-T) domain. The spectrograms in F-T domain are obtained from a set of power spectra of the seismograms with moving time windows.

Before calculating power spectra, all seismograms are normalised with the maximum amplitude of the observed or the full synthetic seismograms. The power spectra are then estimated using the Maximum Entropy Method (MEM), which is well known to provide high resolution spectral estimates even with short time windows (Lacoss, 1971). The length of the time windows are chosen to be at least twice as long as the longest period of interest. Examples of the seismograms and their spectrograms are shown in Fig 3.4. These are calculated for the best-fit waveforms obtained from a synthetic test described in section 3.4. Such spectrograms in F-T domain are used for defining a measure of waveform fit and relative power of a mode-branch in the following.

We quantify both the fit between synthetic and observed waveforms, $\mathbf{f}(\omega, t)$, and the relative power of *j*-th mode, $\mathbf{p}_j(\omega, t)$ in a similar fashion to van Heijst & Woodhouse (1997), but we use spectrograms which can provide a direct estimate of the waveform fit and the relative power in the F-T domain. Both \mathbf{f} and \mathbf{p}_j are matrices. The components of the measure of waveform fit are defined as,

$$f^{kl}(\omega,t) = f(\omega_k,t_l) = \exp\left\{-\frac{\left|S^{obs}(\omega,t) - S^{syn}(\omega,t)\right|}{S^{syn}(\omega,t)}\right\},\tag{3.7}$$

where the indices k and l indicate the frequency and time component, respectively. f^{kl} becomes 1 when the amplitude of spectrogram for observed and synthetic waveforms are identical, whilst it goes to 0 when the misfit between these two spectrograms becomes large.

The components of the measure of relative power of the *j*-th mode, \mathbf{p}_j , are defined as:

$$p_j^{kl}(\omega,t) = p_j(\omega_k,t_l) = 1 - \exp\left\{-W_j(\omega,t)\frac{\left|S^{syn}(\omega,t) - \bar{S}_j^{syn}(\omega,t)\right|}{\bar{S}_j^{syn}(\omega,t)}\right\},\tag{3.8}$$

where $W_j(\omega, t)$ is a weight function which works as F-T domain filter to suppress the contribution from other modes, and is defined as the spectrogram of the *j*-th mode-branch seismogram normalised by its maximum value, $\max[S_j^{syn}(\omega, t)]$



Fig. 3.4. Examples of observed, full synthetic, 1st-higher mode synthetic and residual seismograms for the best-fit model derived from a synthetic test (test I) in Fig 3.6. Spectrograms are shown below the corresponding seismograms, and are used in the reliability analysis in Fig 3.5.

3.4 Synthetic tests

$$W_j(\omega, t) = S_j^{syn}(\omega, t) / \max[S_j^{syn}(\omega, t)].$$
(3.9)

 p_j^{kl} becomes 1 when the *j*-th mode dominates the spectrogram and there is no contribution from the other modes, and becomes 0 if the *j*-th component does not contribute to the spectrogram at all.

The reliability of the measurement can be calculated from the inner product of rows of the two matrices f^{kl} and p_j^{kl} at each frequency,

$$r_j(\omega) = n_t \sum_l p_j^{kl} f^{kl}, \qquad (3.10)$$

where n_t is a normalisation factor for the reliability parameter. Since the reliability is estimated by summing up the product of relative power and waveform fit within a finite time window, the estimated values depend on the length of chosen window. In this study, we chose n_t so that the reliability becomes 1 when both p_j and f are 0.7 for 30 seconds. This criterion can be interpreted as equivalent to perfect relative power and waveform fit (both p_j and f are 1.0) for a 15 second time window. The maximum value of reliability depends on the length of the time span in which waveforms are matched and a particular mode branch is sufficiently energetic.

An illustration of the waveform fit $\mathbf{f}(\omega, t)$, relative power $\mathbf{p}_j(\omega, t)$ and reliability $r_j(\omega)$ are displayed in Fig 3.5 for the results of the synthetic test shown in Fig 3.6. In this example, the waveform is almost completely recovered and the relative power of the fundamental mode is almost 1 for 300 seconds in a time window around 0.01 Hz. This results in the very high values (over 20) of the reliability parameter, because of the normalisation of the reliability so that it is 1 when perfect recovery is achieved for just a 15 second time span.

3.4 Synthetic tests

The non-linear inversion technique using NA has been applied to synthesised seismograms of both Rayleigh and Love waves. Synthetic tests have been performed with a uniformly perturbed 1-D model which contains -5% shear wavespeed perturbation from the PREMC model between Moho and 400 km. This perturbed model is used as a true model whereas the PREMC model is used as a reference model for all the synthetic tests in this section. Two types of tests are carried out using different parameterisations. We used 16 Bsplines ($M_1 = M_4 = 1, M_2 = 10, M_3 = 4$) for the first test (test I) and 12 B-splines ($M_2 = 6$ and the others are the same as test I) for the second test (test II). An event in the Vanuatu region with a depth of 171.4 km is used, and waveforms are calculated for a station NWAO in south-western Australia for which the epicentral distance is 48.7 degree. The input seismograms for a true model (PREMC -5%) are calculated exactly.



Fig. 3.5. Diagrams of the waveform fit $\mathbf{f}(\omega, t)$, the relative power in the *j*-th mode $\mathbf{p}_j(\omega, t)$ and the reliability parameter $r_j(\omega)$ for the first five mode-branches calculated from the results of test I in Fig 3.6.

3000 S-velocity models are generated by NA for each test and we measure phase speed perturbations obtained from the model with minimum misfit.

3.4.1 Rayleigh waves

The results of two synthetic tests for Rayleigh waves are shown in Figs. 3.5 and 3.6. Diagrams of waveform fit and relative power for the Rayleigh-wave tests (Fig 3.5) are calculated from the input seismogram and the best-fit synthetic seismogram for the test I. In the diagram of relative power for each mode branches, we can see that the fundamental mode is well separated from the higher modes. This fact results in a higher relative power for the fundamental mode which further leads to the higher reliability in the phase speed measurement. The first and second higher modes are also well separated from the other modes and they also have higher reliabilities.

The final waveform and the input observation match well and the overall features of the 1-D models are well retrieved for both test I and test II (Fig 3.6). The best-fit waveforms for these tests are almost identical, although the retrieved models show slight discrepancies.

In Fig 3.7 (left column), phase speeds as well as the reliability of the first five mode branches measured from the best-fit 1-D model for both tests are shown. The errors of the phase speed measurements are estimated from the standard deviation of dispersion curves for the best 1000 models. The true perturbation of phase speed is almost completely recovered by the inversion especially for the frequency ranges where the estimated reliability is high enough. If the reliability is low, for example below 0.013 Hz in the 4th-higher mode, we can see some differences between the true and the retrieved phase speeds. Although the retrieved path-specific 1-D models have differences in some depth ranges as seen in Fig 3.6, the estimated phase speed perturbations for these models are almost identical which can be expected from the very good correlation of the best-fit waveforms.

3.4.2 Love waves

The same synthetic tests are also applied to Love wave case. Diagrams of the relative power for several mode branches (Fig 3.8) show that the higher mode branches as well as the fundamental mode overlap in a wide time interval. This reduces the relative power of the fundamental and higher mode Love waves, and results in lower reliabilities than for Rayleigh waves. This overlap of the fundamental and higher mode Love waves also makes it difficult to analyse Love wave dispersion accurately.

The differences of the retrieved 1-D models for Love wave tests (Fig 3.9) are much clearer than those for Rayleigh wave, although both models with minimum misfit are quite close



Fig. 3.6. Results of NA inversion for a synthesised Rayleigh wave. Density plots of the 3000 shear wavespeed models derived from synthetic tests with a -5 % perturbed model are shown in left column. The models are ranked in order of increasing misfit. The best fit model is drawn with a solid white line, the reference model with a dashed white line and the true model with a dotted white line. The number of parameters used in test I is 16 ($M_1 = M_4 = 1$, $M_2 = 10$ and $M_3 = 4$) and in test II is 12 ($M_2 = 6$ and others are the same as in test I). The initial and final fits for the waveforms are displayed in the right column together with diagrams of the fit in the F-T domain. Synthetic waveforms for the true model are drawn with a dotted line and for the best-fit models with a solid grey line.



Fig. 3.7. Phase speeds and the reliability parameters of the first five mode branches measured from the best-fit 1-D models shown in Figs. 3.6 and 3.9. Error bars are estimated from standard deviations of the best 1000 models.



Fig. 3.8. Same as Fig 3.5 but for the results of test I in Fig 3.9.



Fig. 3.9. Same as Fig 3.6 but for a synthesised Love wave.

to the true model. The waveforms calculated for these models are still very similar and the shape of the input waveform is almost completely recovered.

Phase speeds measured from the best fit models are shown in the right column of Fig 3.7. As seen in the case of Rayleigh waves, the true phase speed perturbation is reconstructed



Fig. 3.10. Ray paths for the source-receiver pairs used in the comparative inversions. Five events in the Banda Sea region and four in the Kermadec region are used, chosen from clusters within a $2^{\circ} \times 2^{\circ}$ zone.

very well for the frequency ranges with high reliabilities. Despite the significant differences between the 1-D models retrieved with different parameterisations, the estimated phase speeds are almost identical. This suggests that we can achieve stable multi-mode phasespeed measurements for sufficiently energetic mode branches in a seismogram even with overlapping mode contributions. The tests also demonstrate that the reliability parameter will be a very useful indicator for evaluating measured phase speeds.

From the results of the synthetic tests, we may say that any 1-D model can be a good representation for multi-mode phase speeds as long as synthetic waveforms are well matched to observations. The results strongly support our assumption that path-specific 1-D models are a good representation of multi-mode phase dispersion rather than a direct representation of an actual Earth model. It is also important to note that, even though the path-specific 1-D models are non-unique, this does *not* mean that the existing 3-D models based on such 1-D models are unreliable. We would be able to extract some robust information on the 3-D structure from an ensemble of the 1-D models for a number of paths, even if each 1-D model has some degree of non-uniqueness.

3.5 Application to observed seismograms

The nonlinear inversion method is illustrated by application to two sets of surface wave paths in the Australian region so that we can assess the results of inversion by comparisons of similar paths. Seismic events in very small areas (within $2^{\circ} \times 2^{\circ}$) in the Banda Sea

Event (Harvard Catalog)	lat.	lon.	depth (km)	scalar moment (dyne \cdot cm)
041991D	-6.93	129.51	113.0	1.88×10^{24}
101591D	-6.52	130.07	146.0	2.05×10^{24}
012093H	-7.24	128.60	33.0	4.91×10^{24}
100593B	-6.14	128.92	37.0	4.03×10^{24}
$122595\mathrm{E}$	-6.94	129.18	150.0	4.70×10^{24}
111196A	-32.54	-179.05	33.0	1.67×10^{24}
100797B	-31.84	-178.32	33.0	6.69×10^{24}
122598B	-33.05	-179.32	78.2	2.90×10^{24}
091099F	-32.83	-178.27	33.0	6.37×10^{24}

Table 3.1. Seismic events used for the data inversion with two cluster of events within $2^{\circ} \times 2^{\circ}$ regions in the Banda Sea and near Kermadec Island.

region in Indonesia and near Kermadec Island are chosen (Table 3.1). The NA inversions are performed for two sets of paths, from Banda Sea to CAN station in south-eastern Australia and from Kermadec to TAU station in Tasmania (Fig 3.10).

We first check the radiation pattern from the source using Harvard CMT solutions (e.g., Dziewonski et al., 1981), and seismograms which are near the nodal direction of Love or Rayleigh waves are discarded. Following the criteria of Lebedev (2000) for reducing uncertainties in the phase speed measurements using CMT solutions, the threshold values for the nodal direction have been determined so that the radiation amplitude is less than half of the maximum surface wave radiation. An appropriate reference model is constructed by perturbing the shear wavespeed in the uppermost mantle of PREM or PREMC model, as explained in section 3.2.4. The instrument response is deconvolved from each of the waveforms. The NA waveform inversion is then applied to the observation using the reference model adapted to the data. The frequency bands for the multiple band-pass filters used in the data inversions are 8-12, 10-15, 12-18, 15-22 and 18-25 mHz. Over 3000 1-D shear velocity models are obtained from each path inversion and we select the model with minimum misfit for calculating multi-mode phase speeds of surface waves. The best 1000 models are used to estimate standard errors of the dispersion measurements.

3.5.1 Continental path: Banda Sea to CAN

The paths from the Banda Sea to the CAN station mainly pass through the Australian Continent where we can expect fast wavespeeds as revealed in recent tomography models for the Australian region (e.g., Simons et. al., 1999; Debayle & Kennett, 2000a). The results of the waveform fits and the corresponding diagrams of fit in F-T domain are



Fig. 3.11. Final waveform fits and diagrams of the fit in the F-T domain for the paths from the Banda Sea region to the CAN station.

shown in Fig 3.11. In most cases, the waveform fits are fairly good for fundamental-mode Rayleigh waves. The waveform match is also quite good for Love waves below 0.015 Hz, but gets worse for higher frequencies around 0.02 Hz. Surface waves passing through thick continental crust tend to be contaminated by the effects of strong scattering especially for



Banda Sea Events: CAN station

Rayleigh wave

Fig. 3.12. Density plots of 3000 1-D shear wavespeed profiles obtained from NA inversion for the waveforms shown in Fig 3.11. The best-fit model is drawn with a solid white line and the reference model with a dashed line.

higher frequencies and Love waves are more sensitive to such shallow structures. Thus, the discrepancies at the higher frequencies can be ascribed to the effect of strong scattering



Fig. 3.13. Phase speeds and the reliability parameters for the first three mode branches measured from the best-fit 1-D profiles in Fig 3.12. Error bars are estimated from standard deviations of the best 1000 models.

caused by the thick continental crust of Australia which cannot readily be explained by ray theoretical synthetic waveforms.

Path-specific 1-D models for the paths from Banda Sea to CAN station are displayed in Fig 3.12. A prominent feature of these models is that they all have high velocities in the depth range from 100 to 250 km. These high shear wavespeed are well correlated with a 3-D model of this region (Debayle & Kennett, 2000a). The high wavespeed zone in the 1-D models tends to be thicker for Love wave models which is also consistent with the tomographic models for the region with larger SH wavespeed anomalies than SV wavespeeds (Debayle & Kennett, 2000b). Although the overall features of the retrieved models are similar, there still remain some discrepancies amongst these 1-D models especially in the deeper part of the mantle (below 300 km) which in general cannot be well resolved by surface waves because of the exponential tails of the surface wave eigenfunctions.

Note that such an ambiguity in the 1-D models at depth where there is little sensitivity can be somewhat reduced by introducing an appropriate *a priori* information, as used in most linearised methods of inversion. However, pursuing a realistic and stable Earth model is not our objective, and the 1-D models are just used to compute the phase dispersion, we, therefore, do not apply very strong *a priori* constraints on the model parameters.

The estimated phase speeds for Rayleigh and Love waves are shown in Fig 3.13. For the fundamental mode, the phase speeds of both Rayleigh and Love wave are considerably faster than those for the PREMC model over the frequency range with a high reliability measure, whereas the higher mode Rayleigh waves do not show such a strong velocity perturbation from the PREMC results. The higher mode surface waves are sensitive to variations over a wider range of depth in the 1-D models and do not just reflect local heterogeneities contained in the upper part of the mantle. As a result, the higher mode phase speeds usually have smaller perturbation from the reference model than the fundamental mode. The higher mode Love waves also shows smaller perturbation from the PREMC model for those frequency ranges with a high reliability measure.

3.5.2 Oceanic path: Kermadec to TAU

We further apply the NA inversion approach to oceanic paths from the Kermadec region to the TAU station in Tasmania, for which most regional and global tomography models suggest rather low shear wavespeeds. The final synthetic waveforms are fairly well matched to the observations (Fig 3.14). Unlike the continental paths, the waveform fits for higher frequency Love waves are as good as for lower frequencies.

The path-specific 1-D models in this region (Fig 3.15) show a significant low shear wavespeed zone between 100 to 200 km depth, and they are consistent with the existing



Fig. 3.14. Same as Fig 3.11 but for the paths from the Kermadec region to the TAU station.

3-D model in this region (Debayle & Kennett, 2000a). The deeper parts of these models especially below 300 km are rather contradictory. This is due to the lack of sensitivity to the deep structure since the events used in this region are quite shallow (mainly nominal 33 km) and do not excite enough energy in the higher modes, which are essential to



Kermadec Events: TAU station





Fig. 3.15. Same as Fig 3.12 but for the waveforms shown in Fig 3.14.

reconstruct deeper structure. It should be emphasised that even though the 1-D models differ, the waveforms in Fig 3.14 are fitted to a comparable level.

Phase speeds measured from the 1-D models (Fig 3.16) for this region are consistently slower than the PREM model for both Rayleigh and Love waves. The perturbation of



Fig. 3.16. Same as Fig 3.13 but from the best-fit 1-D profiles in Fig 3.15.
3.6 Discussion

the phase speed from the PREM model is larger for the Rayleigh waves than for Love waves. This also implies a faster SH wavespeed than the SV wavespeed in this region, which is also seen in the polarization anisotropy model of Debayle & Kennett (2000b). The measured phase speeds from the different 1-D models agree well for the first few modes, for the frequency ranges with a high reliability measure. We therefore see again the effectiveness of deriving multi-mode dispersion measurements from the best-fit 1-D profile of shear wavespeed.

3.6 Discussion

We have proposed a new technique to measure multi-mode dispersion from a single seismogram using nonlinear waveform inversion with the Neighbourhood Algorithm. The NA technique itself is based on a simple concept and is quite effective in providing an intensive search of parameter space for models with smaller misfit. The waveform inversion based on this global search technique allows us to find models which fit the waveforms well and makes it possible to measure multi-mode phase speeds accurately. This style of inversion lets us treat both Love and Rayleigh waves independently using isotropic 1-D models. A data adaptive procedure has been developed to find an appropriate reference model for the perturbation analysis used in the synthetic seismogram calculations. The concept of dispersion measurement from a path-specific 1-D profile is quite simple but the technique is found to be very powerful.

It is worth noting that we did not apply strong *a priori* constraints on the 1-D models, which results in some ambiguity in wavespeed. This is because our objective was not to explicitly find a 1-D Earth model, but to extract a better waveform fit and, consequently, a better estimate of phase dispersion. We are able to display the prominent feature of the NA method, i.e., searching for the entire model space to find an ensemble of acceptable models including global minima. Such minima are extremely difficult to find with a conventional linearised inversion technique.

Even though the use of NA as a global search engine requires significantly more computation than linearised inversion methods, our technique has a capacity to analyse around 8,000 seismograms within a month using a Compaq Alpha workstation with 500MHz processor. This may not be as fast as the mode stripping technique (van Heijst & Woodhouse, 1997) but still efficient enough to analyse a large data set for regional scale studies. One of the advantages of the method that we have proposed in this chapter is that surface waves with shorter epicentral distances can be used to measure multi-mode phase speed. This could not be achieved by any other single-station technique for phase speed measurement.

Synthetic tests clearly show that even if there is a substantial overlap of several mode

3.6 Discussion

contributions in a chosen time window, our fully non-linear approach can treat them in a proper manner. The reliability measures for the dispersion measurement can be a great help when we assess the retrieved phase speed, and can also be used as weighting factors for data in inversions for phase speed maps.

There still remain several aspects of the inversion that can be improved. For example, allowing the Moho depth to change during the inversion will be crucial if we treat higher frequency regional phases (more than 30 mHz), although working with such high-frequency ranges, which will require us to consider effects of scattering or mode coupling, is beyond the scope of the current work. In this study, the amplitude term has been fixed and no variation in Q or eigenfunctions has been considered. Such an amplitude variation could also be incorporated in the current technique, with an increase in the number of parameters, which have to be considered in the NA inversion. In order to analyse a large data set with thousands of paths, it would also be desirable to automate all the process of inversion. One of the possibilities for an automated procedure for waveform fitting is presented by Lebedev (2000) and his technique can also be applied with our inversion scheme.

The multi-mode phase speed measured using the new technique can be used to reconstruct phase speed maps for each frequency and for each mode. This would represent a linear inverse problem when we assume great-circle propagation. The phase speed distributions can be further improved in an iterative fashion incorporating ray tracing for the different surface wave modes. Such analysis based on the phase speed measurement allows us to incorporate different types of information, such as polarization anomalies (Laske & Masters, 1996; Yoshizawa, Yomogida & Tsuboi, 1999) and finite-frequency effects of surface wave propagation that will be investigated in the next chapter. Phase speed maps obtained using such multiple information sources for the higher modes as well as the fundamental mode will lead us to more precise images of 3-D Earth models with enhanced vertical resolution.

The influence zone for surface wave paths

4.1 Introduction

Geometrical ray theory has played a major role in many seismological studies, especially in seismic tomography, because of its simple and efficient description of seismic wave propagation, although there are crucial limitations in the ray theory.

One of the well known deficiencies of ray theory is that the theory tends to break down in the presence of strong lateral heterogeneity whose scale-length is comparable to the wavelength of the waves. Wang & Dahlen (1995a) have obtained an empirical condition for the validity of surface-wave ray theory by comparing phase, arrival angle and amplitude anomalies obtained from WKBJ approximation and those from coupled-mode theory. Their condition is derived from a simple assumption that the width of the first Fresnelzone should be much smaller than the scale-length of lateral heterogeneity. This statement has been implicitly recognised since the early stage of the surface wave studies based on geometrical ray theory, and the assumption of smoothly varying heterogeneity around a ray path has been an essential part of ray theory for surface waves (e.g., Woodhouse, 1974; Yomogida, 1985; Tromp & Dahlen, 1992a,b).

One of the ways to overcome the limitations of ray theory is to use scattering theory for surface waves based on the first Born approximation (Snieder, 1986, 1987; Yomogida & Aki, 1987). For body waves at finite frequency, sensitivity kernels for travel times or waveforms have been proposed by many researchers for 2-D cases in the early 90's based on the scattering theory (Luo & Shuster, 1991; Woodward, 1992; Yomogida, 1992; Vasco & Majer, 1993; Li & Tanimoto, 1993; Li & Romanowicz, 1995; Marquering & Snieder, 1995). Such scattering studies have been extended to diffraction studies for 3-D waveform inversion (Meier et al., 1997) and to the construction of 3-D sensitivity kernels (Marquering et al., 1998, 1999; Dahlen et al., 2000; Hung et al., 2000; Zhao et al., 2000). One of the 66

4.1 Introduction



Fig. 4.1. A schematic illustration of influence zone for (a) geometrical ray and for (b) physical ray.

features of these techniques is that they involve an integral over a finite region, whereas the geometrical ray theory is able to treat the velocity variations only along the ray path. Studies of surface-wave scattering based on the Born approximation can be quite useful when local strong heterogeneity exists around a ray path; although the conditions for the application of such a first-order scattering theory to the real Earth may be rather restrictive.

In geometrical ray theory based on the high-frequency approximation, the influence zone around a surface wave path is supposed to resemble a delta function (Fig 4.1). However, actual surface waves with finite frequency should sample a finite region around a ray path. Such a ray with finite width can be termed as a physical ray (Červený & Soares,1992). In this chapter, we focus on determining the effective width of surface wave rays, which can be defined as the *influence zone* around a surface-wave ray path, in which surfacewave phases are coherent and there are only constructive interferences from scattered waves. As an extension of the ray theory, this zone can be found by considering a bundle of neighbouring rays around a central ray path. We should note that the objective of this chapter is to consider a region in which surface waves are coherent and, as a result, we cannot distinguish waves with a slight deviation due to scattering from a true ray. Thus obtaining rigorous sensitivity kernels is beyond the scope of this study, but will be discussed in chapter 7.

In order to investigate the behaviour of rays and to define a particular region surrounding a ray path, we first develop a hybrid ray tracing technique, *Fresnel-area ray tracing* (FRT) for surface waves on a spherical Earth. The concept was originally developed by Červený & Soares (1992) for body waves. The FRT technique consists of two standard ray tracing techniques, kinematic ray tracing (KRT) and dynamic ray tracing (DRT). KRT is used to determine the ray trajectories (geometrical rays) in heterogeneous structures, and DRT provides us with the relative behaviour of neighbouring or paraxial rays. Combining the solutions from KRT and DRT, a paraxial Fresnel area around a ray path can be obtained. In order to trace frequency-dependent surface wave rays in a laterally heterogeneous structure, we need to evaluate surface-wave phase speeds at each geographical point, depending on frequency and mode. Therefore, in this study, we restrict our attention to phase speed structure rather than a 3-D structure, because it is more efficient to work with off-great-circle propagation. FRT makes it possible to construct the stationary-phase field around a ray path (rather than just the great-circle) in a laterally heterogeneous structure. The influence zone around a ray path is estimated from the properties of this stationary-phase field.

4.2 Formulation for Fresnel-area ray tracing

Since the theory and procedure of surface-wave ray tracing, especially KRT and DRT, have been well established by efforts of many researchers (e.g., Woodhouse, 1974; Jobert & Jobert, 1983, 1987; Yomogida & Aki, 1985; Dahlen & Tromp, 1998), we briefly summarise the essence of these standard ray methods. FRT for surface waves on a spherical Earth is then developed by combining the solutions from KRT and DRT.

4.2.1 Kinematic ray tracing

The KRT equations in a spherical polar coordinate system (θ, ϕ) can be represented as a set of three coupled ordinary differential equations (e.g., Aki & Richards, 1980; Dahlen & Tromp, 1998),

$$\frac{d\theta}{ds} = \cos\zeta,\tag{4.1}$$

$$\frac{d\phi}{ds} = \frac{\sin\zeta}{\sin\theta},\tag{4.2}$$

$$\frac{d\zeta}{ds} = \sin\zeta \ \partial_{\theta} \ln c - \frac{\cos\zeta}{\sin\theta} \ \partial_{\phi} \ln c - \cot\theta \sin\zeta, \tag{4.3}$$

where the dependent variable s is the angular distance along a ray path, ζ is the local azimuth which corresponds to the propagation direction of a ray and c is the local phase speed. The geometrical configuration is displayed in Fig 4.2.

When we trace a ray on a spherical Earth, it is convenient to rotate the coordinate system for the source and receiver pairs so that the great-circle lies on the equator as seen in Fig 4.2. In the rotated spherical coordinate system, the source location is always



Fig. 4.2. A surface-wave ray in a "rotated" spherical-polar coordinate system where the source and receiver are on the equator. The propagation direction ζ at (θ, ϕ) is measured from the south.

 $(\pi/2, 0)$ and the coordinate of a receiver at epicentral distance Δ is $(\pi/2, \Delta)$. In this study, all ray tracing is considered in this rotated spherical polar coordinate.

The sets of differential equations (4.1)-(4.3) can be solved numerically with appropriate initial conditions for each equation. In the rotated coordinate system the initial conditions are

$$\theta(0) = \frac{\pi}{2}, \quad \phi(0) = 0, \quad \zeta(0) = \zeta_0.$$
(4.4)

When we calculate an arbitrary ray path with a certain take off angle ζ' at the source, we do not need to estimate the initial angle and we can just put $\zeta_0 = \zeta'$. For a two-point shooting problem, the initial angle $\zeta_0 = \pi/2 + \delta\zeta$ in the rotated coordinate system can be estimated by ray perturbation theory (Woodhouse & Wong, 1986; Dahlen & Tromp, 1998). The perturbed initial take-off angle can be found from

$$\delta\zeta = -\frac{1}{c\sin\Delta} \int_0^\Delta \sin(\Delta - \phi) \partial_\theta \delta c \, d\phi.$$
(4.5)

The integration in (4.5) is to be calculated along the great-circle. If the lateral heterogeneity is not too strong, the linear relation (4.5) offers a fairly good estimate for the initial angle. To hit the receiver, we need to solve the set of ray tracing equations iteratively. A practical numerical scheme for solving these equations is provided by iterative use of the Runge-Kutta method.

4.2.2 Dynamic ray tracing

The kinematic ray tracing systems are useful for tracing an actual ray, but only provide us ray trajectories. When we are interested in the wavefield surrounding a ray rather than just on a line (or ray trajectory), it is necessary to consider the behaviour of the neighbouring or paraxial rays surrounding the central ray. In order to investigate the relative behaviour of rays, we can obtain dynamic ray tracing equations by differentiating the kinematic ray tracing equations (4.1)-(4.3) with respect to the initial take-off angle ζ_0 ,

$$\frac{d}{ds} \left(\frac{\partial \theta}{\partial \zeta_0} \right) = -\sin \zeta \ \frac{\partial \zeta}{\partial \zeta_0},\tag{4.6}$$

$$\frac{d}{ds}\left(\frac{\partial\phi}{\partial\zeta_0}\right) = -\frac{\cot\theta\sin\zeta}{\sin\theta} \frac{\partial\theta}{\partial\zeta_0} + \frac{\cos\zeta}{\sin\theta} \frac{\partial\zeta}{\partial\zeta_0},\tag{4.7}$$

$$\frac{d}{ds}\left(\frac{\partial\zeta}{\partial\zeta_0}\right) = A\frac{\partial\theta}{\partial\zeta_0} + B\frac{\partial\phi}{\partial\zeta_0} + C\frac{\partial\zeta}{\partial\zeta_0},\tag{4.8}$$

where

$$A = \sin\zeta \ \partial_{\theta}^2 \ln c + \frac{1}{\sin\theta} \left[\cot\theta \cos\zeta \ \partial_{\phi} \ln c - \cos\zeta \ \partial_{\theta} \partial_{\phi} \ln c + \frac{\sin\zeta}{\sin\theta} \right], \tag{4.9}$$

$$B = \sin\zeta \ \partial_{\phi}\partial_{\theta}\ln c - \frac{\cos\zeta}{\sin\theta} \ \partial_{\phi}^{2}\ln c, \tag{4.10}$$

$$C = \cos\zeta \ \partial_{\theta} \ln c + \frac{\sin\zeta}{\sin\theta} \ \partial_{\phi} \ln c - \cot\theta \cos\zeta.$$
(4.11)

The initial conditions for these three differential equations may be given by assuming a point source,

$$\frac{\partial\theta(0)}{\partial\zeta_0} = 0, \qquad \frac{\partial\phi(0)}{\partial\zeta_0} = 0, \qquad \frac{\partial\zeta(0)}{\partial\zeta_0} = 1.$$
(4.12)

The geometrical spreading J can be evaluated from the solutions of the DRT equations (4.6)-(4.8) as follows (e.g., Yomogida & Aki, 1985; Jobert & Jobert, 1987),

$$J(s) = \left[\left(\frac{\partial \theta}{\partial \zeta_0} \right)^2 + (\sin \theta)^2 \left(\frac{\partial \phi}{\partial \zeta_0} \right)^2 \right]^{1/2}.$$
(4.13)

If there exist caustics where neighbouring rays cross then

$$\frac{\partial \theta}{\partial \zeta_0} = \frac{\partial \phi}{\partial \zeta_0} = 0, \tag{4.14}$$

and this condition can be used to determine the locations of caustics.



Fig. 4.3. Ray centered coordinate system. An arbitrary point (s, n) is measured from a point on a central ray (s, 0).

4.2.3 Paraxial Fresnel area

Using the results from the KRT and DRT, the Fresnel area surrounding a ray can be estimated based on the paraxial ray theory. The estimated Fresnel area may be called the "paraxial Fresnel area". The theory of paraxial ray approximations has been extensively discussed by Červený (e.g., Červený, 1985, 1987; Červený et al., 1988) for the body wave case and by Yomogida (Yomogida, 1985, 1988; Yomogida & Aki, 1985) for the surface wave case, working in ray-centered coordinates.

First let us introduce a ray-centered coordinate system on a spherical surface (s, n): s corresponds to the distance along the ray path, and n is a coordinate perpendicular to the ray path and n = 0 on the central ray (Fig 4.3). When we expand the phase ψ in a Taylor series around a point on the ray (s, 0) at fixed s,

$$\psi(s,n) = \psi(s,0) + \frac{1}{2}n^2 M(s), \tag{4.15}$$

where,

$$M(s) = \left. \frac{\partial^2 \psi(s,n)}{\partial n^2} \right|_{n=0} = \frac{\omega}{c(s)J(s)} \frac{dJ(s)}{ds}.$$
(4.16)

See Appendix A for explicit formulations of (4.15) and (4.16).

Now let us define the first Fresnel zone surrounding a ray trajectory for a point source at A and a receiver at B (Fig 4.4). Introducing a point F near the ray path, the first Fresnel zone is defined in terms of the phase behaviour as follows,

$$\left|\psi_A^F + \psi_B^F - \psi_A^B\right| \le \pi,\tag{4.17}$$

where ψ_A^F , ψ_B^F and ψ_A^B are the phases integrated along ray paths A - F, B - F and A - B.



Fig. 4.4. Schematic view of Fresnel area around a ray. The radius of Fresnel zone is defined by the line $F-O_F$.

Considering the points F at (s, n) and O_F at (s, 0), the phases ψ_A^F and ψ_B^F can be obtained from (4.15),

$$\psi_A^F = \psi_A^{O_F} + \frac{1}{2} n^2 M_A^{O_F}, \tag{4.18}$$

$$\psi_B^F = \psi_B^{O_F} + \frac{1}{2} n^2 M_B^{O_F}.$$
(4.19)

Both (4.18) and (4.19) are defined along the same ray trajectory, but in different raycentered coordinate systems. That is, the coordinate system of (4.18) is (s, n), whilst (4.19) is $(\Delta - s, n)$, where Δ is the distance from source to receiver along the ray path. Using the relation $\psi_A^{O_F} + \psi_B^{O_F} = \psi_A^B$, and substituting (4.18) and (4.19) into (4.17), the equation for the paraxial Fresnel area can be obtained,

$$\frac{1}{2}n^2 \left| M_A^{O_F} + M_B^{O_F} \right| \le \pi.$$
(4.20)

We can finally get the radius of the paraxial Fresnel area measured from the ray path,

$$n = \left[\frac{2\pi}{|M_A^{O_F} + M_B^{O_F}|}\right]^{1/2},\tag{4.21}$$

where $M_A^{O_F}$ and $M_B^{O_F}$ can be expressed as

$$M_{A}^{O_{F}} = \frac{\omega}{c(O_{F})} \frac{J_{A}'(O_{F})}{J_{A}(O_{F})}, \qquad M_{B}^{O_{F}} = \frac{\omega}{c(O_{F})} \frac{J_{B}'(O_{F})}{J_{B}(O_{F})}, \tag{4.22}$$

with J' = dJ/ds. On inserting (4.22) into (4.21), the final form for the radius of the paraxial Fresnel area can be expressed as



Fig. 4.5. Illustration of an exact Fresnel area on a spherical surface in a rotated spherical-polar coordinate system.

$$n = \left[\frac{2\pi c}{\omega}K\right]^{1/2} = [TcK]^{1/2} = [\lambda K]^{1/2}, \qquad (4.23)$$

where T is the period of the wave, λ is a wavelength and $K = J_A J_B / |J'_A J_B + J'_B J_A|$. At caustic points, the geometrical spreading J shrinks to 0, and thus the radius of the paraxial Fresnel area is also 0. Since the radius of the exact Fresnel area at the point source is very close to $\lambda/2$ in case of the first Fresnel zone (see Appendix B), we may therefore expect a radius of the Fresnel zone of the same order of $\lambda/2$ even at caustics.

4.3 Synthetic tests of Fresnel-area ray tracing

In order to check the validity of the formulation and behaviour of Fresnel zones in laterally heterogeneous structure, examples of paraxial Fresnel areas in synthetic models are displayed in this section.

4.3.1 Comparison with the exact and the paraxial Fresnel area

The exact Fresnel area can be simply calculated for a laterally homogeneous structure since we can compute the travel time along great-circle without any ray tracing. Let us consider the geometrical configuration in the rotated spherical polar coordinates as shown in Fig 4.5. The definition of the first Fresnel zone, with respect to the path length, can be written as

$$\left|\Delta_A^F + \Delta_B^F - \Delta_A^B\right| \le \frac{\lambda}{2},\tag{4.24}$$

which provides an alternative form to (4.17). Δ_A^B is the epicentral distance and the arclengths Δ_A^F and Δ_B^F can be expressed in terms of spherical trigonometry,

$$\cos \Delta_A^F = \cos \frac{\pi}{2} \cos \theta + \sin \frac{\pi}{2} \sin \theta \cos \phi = \sin \theta \cos \phi, \qquad (4.25)$$

$$\cos \Delta_B^F = \cos \frac{\pi}{2} \cos \theta + \sin \frac{\pi}{2} \sin \theta \cos(\Delta_A^B - \phi) = \sin \theta \cos(\Delta_A^B - \phi).$$
(4.26)

The boundaries of the exact Fresnel area on the spherical surface can be obtained by solving (4.24)-(4.26) numerically.

Examples of the exact Fresnel area and the paraxial Fresnel area in a laterally homogeneous structure for three different periods (25, 40, 100 seconds) are shown in Fig 4.6 (a). We can see that the paraxial Fresnel area is a fairly good approximation of the exact one, except close to the source and receiver positions where the geometrical spreading shrinks to 0 and, as a results, the paraxial Fresnel area vanishes. As the period increases, the agreement between the exact and the approximate Fresnel area becomes worse. As mentioned at the end of the previous section, we may rectify the problem by assuming that the paraxial Fresnel area at the source and the receiver as well as at any possible caustic point have a radius of the Fresnel area of the order of $\lambda/2$ for the first Fresnel zone. Paraxial Fresnel areas with corrections at the source and the receiver are shown in Fig 4.6 (b). Such corrections are quite useful for matching the paraxial Fresnel areas and the exact ones even for longer period. The detailed procedure for the correction of the paraxial Fresnel area is explained in Appendix B.

4.3.2 Hot-spot heterogeneity

We next perform synthetic tests with simple hot-spot models (Fig 4.7). These models contain a circular region whose radius is 4 degrees and the Rayleigh-wave phase speed is 10 % slower than the surrounding area. These tests give us insight into how the location of a strong heterogeneity affects the wavefield and the Fresnel area around a ray.

As can be clearly seen in Fig 4.7, if the source is adjacent to a strongly heterogeneous region, ray paths which are radiated toward the heterogeneity are significantly distorted. When the source lies slightly away from the heterogeneity but still close enough, we can see areas of focusing and defocusing behind the circular heterogeneity. The Fresnel areas shown for each heterogeneity configuration suggest that they are severely affected by the



Fig. 4.6. (a) Exact (solid ellipsoid) and paraxial (shaded ellipsoid) Fresnel areas at 25 (left), 40 (middle) and 100 (right) seconds. Background map is only plotted for the measure of the scale. (b) Same as (a) but for paraxial Fresnel areas with correction at source and receiver.



Fig. 4.7. Ray paths in hot-spot models (left) and corresponding paraxial Fresnel areas for 40 seconds (right). Both regions inside and outside the circle are homogeneous, but the Rayleigh-wave phase speed inside the circle is 10 % slower than the outside. The locations of the hot-spot are 5°S, 140°E (top) and 10° S, 140°E (bottom). Source location is 1°S, 140°E for both case. The distances from sources to the center of the circle are 4° (top) and 9° degree (bottom). Rays are radiated with the azimuths from 90° to 270° for every 3°.

strong velocity gradient in the vicinity of the heterogeneity and, as a results, the radii of Fresnel zone become smaller in such regions because the ray-path density is very high, in other words, the surface-wave energy concentrates in that particular area.

We should note here that, with a ray-based technique used in this study, we cannot



Fig. 4.8. Geometrical configuration of influence zone (shaded area), a geometrical ray (AO_FB) and a perturbed ray (AFB).

treat the full range of complex multi-path effects which can be caused in the presence of a strong lateral heterogeneity like this hot-spot model, although fortunately such rapid and strong velocity variations are not seen in most phase speed models.

4.4 Influence zone inferred from stationary-phase field

In the previous section, we have shown the estimates of the first Fresnel zones obtained from FRT. Now we examine the nature of the variation around surface-wave paths as an extension of the geometrical ray theory using stationary-phase fields.

4.4.1 Stationary phase field

Using FRT, stationary phases around surface-wave ray paths in laterally heterogeneous structure can be simply evaluated. First let us define a reference waveform U_0 along a central ray for a frequency ω as,

$$U_0(\omega) = A_0(\omega) \exp(-i\psi_0(\omega)), \qquad (4.27)$$

where the complex amplitude term A_0 contains the source radiation, the receiver eigenfunction, geometrical spreading and the spatial variation of amplitude caused by a background structure, ψ_0 is phase of the wave integrated along the ray. Explicit forms for A_0 can be found in many text books (e.g., Aki & Richards, 1980; Kennett, 1983; Dahlen & Tromp, 1998). As in most surface wave studies, the wavefield U_0 along a ray path can be calculated as a WKBJ seismogram. Hereafter we abbreviate the notation for the frequency dependency. Now let us assume that the waveforms along perturbed rays which arrive at a receiver passing through a point F(s, n) near the central ray (Fig 4.8) can be written as,

$$U_F = A_F \exp(-i\psi_F),\tag{4.28}$$

where A_F is a perturbed amplitude term for the off-centre ray path and $\psi_F = \psi_A^F + \psi_B^F$ is a perturbed phase term integrated along the path. U_F represents the perturbed wave and is to be distinguished from scattered or diffracted waves for which an inclination factor and different geometrical spreading need to be considered (e.g., Born & Wolf, 1999). From (4.20), the perturbation of phase between a central ray and neighbouring rays can be expressed as,

$$\delta\psi_F = \psi_F - \psi_0 = \frac{1}{2}n^2 M_F(s) \tag{4.29}$$

where $M_F(s) = \left| M_A^{O_F}(s) + M_B^{O_F}(s) \right|$ is derived from FRT. Substituting (4.29) into (4.28) and using (4.27), we obtain a relation between U_F and U_0 ,

$$U_F = A_F \exp(-i\psi_0) \exp(-i\delta\psi_F)$$
(4.30)

$$= \frac{A_F}{A_0} U_0 \exp(-i\delta\psi_F). \tag{4.31}$$

The exponential term in (4.31) depends only on the background structure, and we can therefore estimate the stationary-phase field around a central ray path by evaluating this term.

To investigate the behaviour of the phase term in (4.31), let us consider an explosive source for which the azimuthally dependent radiation effect can be ignored, and assume that amplitude variation in the background structure is smooth and differences of epicentral distance along a central ray and along perturbed rays are small enough so that the differences in geometrical spreading can be ignored. In such circumstances, we can assume that $A_F \approx A_0$ near the central ray path, and the perturbed waveform can be represented as,

$$U_F \approx U_0 \exp(-i\delta\psi_F) = U_0 \exp\left(-\frac{i}{2}n^2 M_F\right).$$
(4.32)

An example of the stationary-phase function $\exp\left(-\frac{i}{2}n^2M_F\right)$ for ray paths to CAN and NWAO stations are represented in Fig 4.9 (a). Surface wave rays are traced on a 40-second Rayleigh-wave phase speed model obtained from a 3-D Australian Continent model of Debayle & Kennett (2000a). Throughout this chapter, this 3-D model is used to reconstruct 2-D phase speed maps. Crustal corrections for these phase speed maps are made by using the 3SMAC model (Nataf & Richard, 1996). Contours around each ray path correspond to the Fresnel zones, and up to the eighth Fresnel zone is shown in this example.



Fig. 4.9. (a) Spatial projections of real-part of stationary-phase function for CAN and NWAO stations. White solid lines are geometrical rays and white dotted lines are corresponding great-circle. Rays are traced on a 40-second Rayleigh-wave phase speed map. (b) Cross section of stationary-phase field along white thick-dotted lines in (a). Solid lines are real part of stationary-phase function and dashed lines are imaginary part. Vertical dashed lines show the first Fresnel zone and shaded areas show 1/3-width of the first Fresnel zone.

The cross sections of the phase function of both real and imaginary parts at middle of the source-receiver distance are shown in Fig 4.9 (b). This type of function around a ray path has previously been presented in the context of Born scattering (Aki & Richards, 1980; Yomogida, 1992; Marquering et al., 1998). Yomogida (1992) obtained Fréchet kernels, which have the similar character to the phase functions shown here, considering Fresnel zones around a ray path for body-wave case. Marquering et al. (1998) used the phase function for a discussion of the validity of the stationary-phase approximation. Note the definition of the Fresnel zone implicitly includes a first-order Born approximation. However, it should be emphasised that our objective is not to try to discuss rigorous sensitivity kernels considering scattering or diffraction, but to focus on the determination of a region around the geometrical ray path for surface waves, in which the surface-wave

phases can be regarded to be coherent, since we wish to understand the scattering region appropriate to surface wave tomography.

4.4.2 Extended influence zone

In geometrical ray theory, the waveform U_0 is evaluated along a ray (or a great-circle). Thus the influence zone for U_0 is supposed to be just like a delta function on the ray (Fig 4.1). We now take a different view point from traditional ray theory to extend the effective zone for the surface wavefield. We want to find the region about the ray (the *influence zone*) in which the wavefield is coherent. An average wavefield U_{av} over a zone Σ can be defined as,

$$U_{av} \equiv \frac{\int \int_{\Sigma} U_F(s_r, 0; s, n) ds dn}{\int \int_{\Sigma} ds dn},$$
(4.33)

where Σ is an area around the ray on the spherical surface and $U_F(s_r, 0; s, n)$ is the waveform along a path which passes through a point (s, n) and reaches the receiver $(s_r, 0)$ (Fig 4.8). Using the paraxial approximation (4.32), the average wavefield can be expressed as,

$$U_{av} \approx U_0 \frac{\int \int_{\Sigma} \exp\left(-i\delta\psi_F\right) ds dn}{\int \int_{\Sigma} ds dn}.$$
(4.34)

Thus if we require the average wavefield U_{av} to be approximately equal to U_0 , we require the average to be taken over an influence zone Σ_I such that,

$$\frac{\int \int_{\Sigma_I} \exp\left(-i\delta\psi_F\right) ds dn}{\int \int_{\Sigma_I} ds dn} \approx 1.$$
(4.35)

The expression (4.35) lead us to a necessary and sufficient condition for the influence zone,

$$\exp(-i\delta\psi_F) = \exp\left(-\frac{i}{2}n^2M_F(s)\right) \approx 1.$$
(4.36)

It is clear that (4.36) is sufficient to satisfy (4.35). The necessity of the condition (4.36) arises from the fact that the denominator in (4.35) is a monotonically increasing real function whereas the numerator is a complex function which is oscillating along the *n*-direction. It may be worth noting that the range of the *n*-integration should not be too far away from the central ray, because we assumed that both the spatial amplitude variation and the differences of epicentral distances between rays should be small in the region under consideration (see (4.32)). Note that the condition (4.36) is also valid for

the geometrical ray theory in which *n*-integration in (4.34) is simply evaluated just on the central-ray, that is, n = 0; this satisfies the condition (4.36) exactly, i.e., $\exp(-i0) = 1$.

The influence zone for the surface wave paths, which satisfies the necessary and sufficient condition (4.36), can be readily found from a diagram of stationary-phase function (Fig 4.9 (b)). We can recognise a fairly flat area around a central ray (n = 0) in the real part of the phase function whilst the imaginary part is close to zero. Now we will focus on this region to determine the influence zone. For this purpose, we use the condition for wavefield coherence in an integral form (4.35) to investigate the zone which may have significant effects on the total wavefield. The condition (4.35) can be rewritten by using (4.29) as,

$$I(n_I) = \frac{\int_0^\Delta \int_{-n_I W(s)}^{n_I W(s)} \exp\left(-\frac{i}{2}n^2 M(s)\right) dn ds}{\int_0^\Delta \int_{-n_I W(s)}^{n_I W(s)} dn ds} \approx 1,$$
(4.37)

where Δ is a ray length, W(s) is a half-width of the first Fresnel zone at a point s on the central ray, and n_I is a coefficient for determining the width of a region to be integrated. n_I is normalised to be 1.0 for the first Fresnel zone. Considering the real and imaginary parts of $I(n_I)$, the condition (4.37) can be reformulated as,

$$\operatorname{Re}\left\{I(n_{I})\right\} \approx 1 \qquad \operatorname{Im}\left\{I(n_{I})\right\} \approx 0. \tag{4.38}$$

Fig 4.10 shows diagrams of the real and imaginary parts of $I(n_I)$ as a function of the normalised coefficient n_I . The criteria can be satisfied exactly only at the central ray path. We, therefore, need to set threshold values which satisfy (4.38) reasonably well such as,

$$\operatorname{Re}\{I(n_I)\} \ge 0.9 \qquad |\operatorname{Im}\{I(n_I)\}| \le 0.1.$$
 (4.39)

At the 1/3-width of the first Fresnel zone (i.e., $n_I = 1/3$), we find that Re $\{I(n_I)\} \approx 0.98$ and $|\text{Im}\{I(n_I)\}| \approx 0.1$ (Fig 4.10), which satisfies the condition (4.39). This area can be regarded as the *influence zone* over which the perturbed waveforms are fairly coherent in phase; giving a constructive interference for surface wavefields within the influence zone, whilst outside there is rather a destructive interference caused by the rapid fluctuation of the phase function, as shown in Fig 4.9 (b).

Since the width of the Fresnel zone is proportional to the square root of the phase differences between the central ray path and a neighbouring ray path (see (4.21)), the condition on the size of the influence zone can be written in a similar form to (4.17),

$$\left|\delta\psi_F\right| = \left|\psi_A^F + \psi_B^F - \psi_A^B\right| \le \frac{\pi}{9}.\tag{4.40}$$



Fig. 4.10. (a) Real part of an integral function $I(n_I)$ as a function of coefficients for a distance from the central ray path n_I . Note that n_I is normalised so that the n_I becomes 1 at the first Fresnel zone. (b) Same as (a) but for the imaginary part of $I(n_I)$. Horizontal dashed lines show the criteria for the influence zone (see (4.39)) and shaded areas show 1/3-width of the first Fresnel zone $(n_I = 1/3)$.

The choice of the 1/3-width of the first Fresnel zone as the *influence zone* may seem to be somewhat arbitrary, but this is a reasonable choice as discussed in the following section in a context of the uncertainties in actual phase speed measurements.

We should recall that the paraxial Fresnel zone shrinks to 0 at source, receiver and caustic points. However, the radius of the first Fresnel zone at these points are about $\lambda/2$. Thus the radius of the influence zone at source and receiver is expected to be $\lambda/6$. A similar argument may be applied along the path and the influence zone can be extended slightly beyond the positions of source and receiver (see Appendix B).

Examples of the *influence zones* for the ray paths in Fig 4.9 (a) are shown in Fig 4.11 for fundamental-mode Rayleigh waves at 40 and 100 seconds. The width of the physical rays become wider for longer period waves, because the width of the Fresnel zone is proportional to the square-root of the product of phase speed and the period of the waves as shown in (4.23). In Fig 4.12, physical rays for both the fundamental and the first higher-mode Rayleigh waves at 40 seconds are displayed. These rays pass through a region with moderate (not smooth) heterogeneities near the continent-ocean boundary in eastern Australia. For the fundamental mode, the great-circle between source and receiver grazes along the edge of the influence zone, whilst the ray for the first higher-mode passes the other side of the great-circle. In such a case, we will need to take into account both the different influence zones as well as the different ray paths for each mode. Such effects cannot be treated with the traditional geometrical ray theory and with the greatcircle approximation. Thus, physical rays in phase speed structures provide us with the possibility to enhance the current methods of surface wave analysis, even with moderate lateral heterogeneity for which geometrical ray theory and the approximation of wave propagation along the great-circle tend to break down.

4.4.3 Evaluation of influence zone

We now discuss the nature of the influence zone defined in the previous section. First let us look at the stationary-phase function in the time domain as a function of distance from a ray. Fig 4.13 displays the phase function in the time domain with a narrow-frequency band around 25 mHz (40 seconds) at $n_I = 0$, $n_I = \frac{1}{3}$ and $n_I = 1$ (where $n_I = 1$ corresponds to the half-width of the first Fresnel zone). The phase function at $n_I = \frac{1}{3}$ is quite coherent with that at $n_I = 0$ with only a very slight phase shift. This small phase shift is equivalent to the maximum differences in arrival time of a wave along a central ray and that along a neighbouring ray within the influence zone. The arrival-time delay within the influence zone can be estimated analytically. The width of the Fresnel zone is proportional to the square root of the period T of the wave (see (4.23)), so that the 1/3 width of the first Fresnel zone depends on 1/9 of the period. Since the first Fresnel zone is defined as the half-period zone, the arrival-time delay $\delta\tau$ for the influence zone can be estimated as,



Fig. 4.11. Physical rays of the fundamental-mode Rayleigh waves for two paths to CAN and NWAO stations at 40 seconds (top) and 100 seconds (bottom) with correction at the source and receiver. Shaded elliptical areas show the influence zone, dashed ellipsoid show the first Fresnel area and dotted lines show corresponding great-circle.



Fig. 4.12. Physical rays of the fundamental mode (black solid ellipsoid) and the first-higher mode (gray shaded ellipsoid) to CAN station. Correction at the source and receiver locations is applied. Geometrical rays of these paths are shown in black (the fundamental mode) and white (the first-higher mode) dashed lines. Rays are traced on a 40-second Rayleigh wave phase speed model. A black dotted line shows the corresponding great-circle.



Stationary-phase function in time domain

Fig. 4.13. Inverse-Fourier-transformed stationary-phase function with a narrow-frequency band around 40 seconds. The phase function in time domain at n = 0 is shown as a solid line, that at $n = 1/3n_1$, where n_1 is the radius of the first Fresnel zone, as a dashed line and that at $n = n_1$ by a dotted line.

 $\frac{T}{2} \times (\frac{1}{3})^2 = \frac{T}{18}$, which is equivalent to around 5.6 % of the period T. For 40-second surface waves, $\delta \tau \approx 2.2$ seconds.

As we have explained in the previous section, the surface wavefields within the influence zone are assumed to be coherent. In other words, we cannot distinguish surface waves along different paths that are passing inside the influence zone. However, phase coherency tends to be violated as the perpendicular distance from the central ray become large. This effect can be investigated considering the differences in a path-average phase speed along the central path and that along a neighbouring path near the edge of the influence zone as follows. Let us consider the phase of a surface wave along the central ray, $\omega X/\hat{c}$, and that along a neighbouring ray, $\omega(X + \delta X)/(\hat{c} + \delta \hat{c})$, where X and \hat{c} are the ray length along the central ray and the corresponding path-average phase speed, respectively. δX and $\delta \hat{c}$ are the differences in ray length and in path-average phase speeds between the central and neighbouring rays. Here, we ignore the effect of the initial phase from the source and only consider the propagation effect on phase, assuming that these phase along different rays within the influence zone are coherent and approximately identical,

$$\frac{\omega X}{\hat{c}} \approx \frac{\omega (X + \delta X)}{\hat{c} + \delta \hat{c}}.$$
(4.41)

If we put $\delta \hat{c} = \varepsilon \hat{c}$, where ε is a small parameter which corresponds to an uncertainty in the perturbation of path-average phase speeds along different ray paths, then, ε can be represented as,

$$\varepsilon = \frac{\delta \hat{c}}{\hat{c}} \approx \frac{\delta X}{X}.$$
(4.42)

For our definition of the influence zone, $\delta X = \frac{\lambda}{2}(\frac{1}{3})^2 = \frac{\lambda}{18}$. Now, let us think about Rayleigh waves at 40-seconds whose phase speed is around 3.9 km/s. A typical epicentral distance in regional tomography is around 3000 km. In this case, from (4.42), the difference in the average phase speed along a central ray and that along a neighbouring ray near the edge of the influence zone is less than 0.3 % of the phase speed. This value is equivalent or less than the errors in measured phase speeds, which supports the validity of the phase coherency within the influence zone. Note that the estimated errors of phase speeds from (4.42) become large for longer periods, because it is proportional to the wavelength. Therefore, the coherency in phase within the influence zone gets worse for longer periods.

The above argument for the validity of the phase coherency also raise a very important aspect of phase speed measurements at finite frequency. The phase speeds of surface waves are generally measured along the great-circle paths and inevitably have some error in measurement. As we have seen, the apparent phase speed changes (4.42) introduced by deviations in ray path within the influence zone will generally be less than the errors in phase speed measurement. For lower frequency surface waves we can therefore regard the measured phase speeds as an average over the influence zone, rather than an average along the great-circle (or the appropriate ray).

Within the influence zone we cannot distinguish scattered waves from bent rays because they have such similar-phase. We can therefore treat the entire influence zone as equivalent when inverting for phase speed maps, and employ an area average of the surface-wave phase over the influence zone. Because this zone is chosen so that there is very little variation in phase, we do not need to employ rigorous calculation of the sensitivity kernels, with considerable computational savings.

However, outside the influence zone the phases are not coherent. Once we include such paths we need to employ full sensitivity kernels around the central ray paths to accommodate the effects of scattering and diffraction. Such sensitivity kernels for phase speed structures are discussed in more detail in chapter 7 based on the Born and Rytov approximations.

The *influence zone* defined in the previous section is based on a number of assumptions. However we can justify the validity of these assumptions in the following way. Since the influence zone is not too wide and lies close to the central ray (Fig 4.11), the azimuthally dependent radiation effects from a double couple source may be ignored in a good approximation, except near a nodal direction. Such small width of the influence zone also helps to justify the assumption of small spatial amplitude variation and also that of small changes in epicentral distances between rays in the zone. We should note that these assumptions

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become worse as the period of waves becomes longer and as the mode-branch number increases, because the width of the influence zone becomes wider for these waves.

The finite-width rays can also be used to estimate the limits on lateral resolution in surface wave tomography. The typical width of physical rays for 40 second Rayleigh wave with epicentral distances 3000 km is around 200 km. If two physical rays cross over, the diagonal spread of the cross-over region should be slightly larger than the width of the influence zone of the rays (Fig 4.11). The estimated lateral resolution for regional surface wave tomography is around 300 km (e.g., Debayle & Kennett, 2000a). Therefore, the scale of lateral resolution of the tomography is fairly consistent with that of the width of the physical rays, indicating the utility of the concept of the influence zone.

4.5 Discussion

By using the FRT technique, we have defined the influence zone for surface waves by considering a bundle of neighbouring rays around a ray path. Our definition of the influence zone is that surface waves are coherent in phase within the zone, which implies that the scattered energy within this zone affects the total surface wavefield in a constructive way. The estimated width of the influence zone is approximately 1/3 of that of the first Fresnel zone.

The influence zone for surface waves defined in this study can be simply applied in 2-D phase speed inversions. In such a case, the phase speeds measured from observations are no longer just a "path average", but can be regarded as "area average" within the influence zone. From the perspective of finding a realistic Earth model, the difference will be a slight blurring of the phase-speed maps. The use of the physical rays is more appropriate than the use of geometrical rays, in that the realistic finite frequency effect of wave propagation can be taken into account in tomographic inversion. Vasco et al. (1995) have applied simple Fresnel volumes for body waves (corresponding to a Fresnel "area" for surface waves) using a similar technique to this study based on the method of Červený & Soares (1992). They have shown that tomographic inversions with the Fresnel volumes provide comparable models with those obtained by using rigorous sensitivity kernels, which requires considerably more numerical computations.

In most studies of 2-D and 3-D sensitivity kernels, there are prominent variations in sensitivities along the path, that is, the maximum sensitivities appear in the vicinity of the source and receiver. Vasco et al. (1995) have also suggested that such variations of the sensitivity along the path can be considered by normalising the Fresnel volume by its elliptical cross-section area perpendicular to the path (corresponding to the width of the Fresnel area in 2-D case). This results in a sensitivity peaked at the source and receiver

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locations. Such an approach can also be applied to the influence zone in this study, when we apply it to inversions for phase speed maps, and provides a means of coping with errors in source location. If locations and neighbouring structure of the source and receiver are fixed, there is then strong latent contributions from the sensitivity kernel, but these do not influence the actual results.

In the application of the influence zone to surface wave tomography, it may be worth applying a weight function perpendicular to the central ray to reduce the errors arising from the slightly larger phase differences near the edge of the influence zone. If scattering effects are conspicuous in the observed waveforms and we need to consider scattering effects outside the influence zone, some rigorous calculations for sensitivity kernels will be necessary to take into account the complex effects of scattering and diffraction, which may interfere rather destructively with the total wavefield. As long as we are working with intermediate period waves (say longer than 40 seconds), strong scattering effects are not expected in the real Earth. However, as the period of interest becomes shorter, we may need to consider the effects of scattering or coupling between mode branches (e.g., Kennett, 1984; Kennett & Nolet, 1990; Marquering et al.,1999). A way to calculate more rigorous sensitivity kernels based on the first-order scattering of surface waves will be discussed in chapter 7.

One of the significant advantages of the FRT technique is that off-great-circle propagation can be treated effectively. Since the width of the Fresnel zone around a ray path depends on the phase-velocity gradient which is evaluated on the central ray, the influence zone should be obtained around an actual ray rather than around a great-circle. However, this may not be critical issue, since the great-circle and the actual ray path are very close each other if the great-circle lies in the influence zone.

The concept of the influence zone also gives us an insight into the validity of the greatcircle approximation which has been widely used in most studies of surface-wave tomography. Since the influence zone is defined so that the waveforms within the area are coherent, we may say that phases of surface waves along rays passing out of the influence zone are no longer coherent with the phase along the central ray path. In other words, if the greatcircle lies outside of the estimated influence zone for a model, the waves along an actual ray and the corresponding great-circle should be significantly different, which may result in mislocation of heterogeneity in tomographic models.

Since the FRT approach relies on paraxial ray theory, which is based on a high-frequency approximation, a velocity structure should not vary appreciably within a width of the Fresnel area. Recent tomography models in regional scales (e.g., Debayle & Kennett, 2000a) have shown quite large velocity variations (over $\pm 10\%$) in the uppermost mantle,

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although these models are still derived from the assumption of wave propagation along the great-circle. Such models with moderate lateral heterogeneities seem to be at the limit of the ray-based technique and the great-circle approximation is about to be violated. Even though our approach is still based on similar limitations to the conventional ray theory, we may slightly extend the limit of the ray-based method by introducing ray tracing and considering the effects from surrounding regions about a surface-wave ray path to take into account the off-great-circle propagation as well as the finite-frequency effects.

The method of FRT is simple and computationally effective and, therefore, allows us to apply it in large-scale inversions. Although the scattered waves coming from outside the influence zone cannot be fully treated with this approach, such scattering effects seem not be so important in the intermediate period range (40-150 seconds). We are now able to deal with ray paths with finite-width as well as the deviations in propagation from the great-circle in a simple, computationally efficient form. The new technique of the multimode dispersion measurements for regional surface waves developed in chapter 3 allows us to reconstruct multi-mode phase speed maps on regional scales. Together with such multimode information and the influence zone for phase speed structures, we can envisage a new approach for reconstructing 3-D image of the upper mantle from multi-mode surface waves, which is further investigated in chapter 5. The concept of the influence zone presented here should be of great help in extending the current methods of surface wave tomography which have commonly been based on geometrical ray theory and on the approximation of great-circle propagation. Three-stage inversion: A new approach for surface wave tomography

5.1 Introduction

Current methods of surface wave tomography are based on multi-stage processes using either the exploitation of fundamental mode dispersion (e.g., Ekström et al., 1997; Ritzwoller & Levshin, 1998) or the multi-mode waveform inversion for a path-specific 1-D model (e.g., Cara & Lévêque, 1987; Nolet 1990).

Global studies generally take the former approach with fundamental mode dispersion, and a number of high resolution phase speed maps for fundamental mode surface waves have been proposed (Trampert & Woodhouse, 1995, 1996; Zhang & Lay, 1996; Ekström et al., 1997). Higher mode information can be also incorporated via mode-stripping technique (van Heijst & Woodhouse, 1997, 1999), although the use of higher modes has been rather limited.

Regional surface wave tomography is, in general, based on two-stage inversion methods. The first stage is multi-mode waveform fitting for a path-average 1-D model. In the partitioned waveform approach formalised by Nolet (1990), the 1-D models obtained by waveform fitting are interpreted as the average structure along the path between source and receiver. The ensemble of path averaged constraints are then used in a linearised inversion to recover final 3-D structure (Zielhuis & Nolet, 1994). The waveform inversion is based on linearised inversion with either direct use of the seismograms (Nolet et al., 1986) or the use of secondary variables based on cross-correlation between observed and synthetic seismograms (Cara & Lévêque, 1987). Both methods show dependence on the reference model used for initiating the inversion. In the second stage, a 3-D model is retrieved from the ensemble of path-specific constraints using linearised inversion as a form of cellular tomography (Zielhuis & Nolet, 1994), or in a continuous representation 91

with a Gaussian smoothing, defined by a correlation length of model parameters based on the continuous regionalisation scheme of Montagner (1986).

The common feature of such different applications is that the path-specific 1-D models obtained in the first stage of the process are interpreted directly as averages along the paths. Marquering et al. (1996) has pointed out that, as frequency increases, this pathaverage assumption has significant limitations for higher mode information representing body waves, since the sensitivity of the data is concentrated around the body wave paths. By considering coupling between the mode branches, improved results for data set with a large higher mode component can be obtained but with considerable increase in computation time.

The more common approach for improving tomography models has been to enlarge the number of paths so that more detailed structure can be recovered with dense path coverage. For example, for the Australian region, Simons et al (1999) and Debayle & Kennett (2000a) have used around two thousand paths in inversions using Rayleigh waves. With such path densities, it is possible to extend the second stage inversion to try to extract azimuthal anisotropy (Debayle & Kennett, 2000a).

The levels of heterogeneity and heterogeneity gradient revealed in recent tomographic studies of the upper mantle are probably too large for the path-average approximation to be applied directly to the 1-D models with the assumption of the surface wave propagation along the great-circle. However, for the frequency range in which modal interaction can be neglected, we can use the path-average assumption for the phase of individual mode contributions and regard the 1-D model as a representation of the character of multi-mode dispersion along the source-receiver path. This viewpoint is reinforced by investigation of fully non-linear inversion for surface wavetrains explained in chapter 3, which demonstrates the possibility of extracting different styles of 1-D models with a comparable fit to data. Although the models differ significantly, the dispersion of the first few modes over the frequency range cannot be distinguished.

The existence of large velocity perturbations in the recent tomography models also warn us to rethink the great-circle approximation for the surface-wave paths as well as the finite-frequency effects of surface wave propagation. In chapter 4, we have studied the approximate zone of influence around surface wave paths with careful investigation of a stationary-phase field around a path. We have shown that the approximate influence zone can be represented as roughly 1/3-width of the first Fresnel zone. The idea of the influence zone leads us to an alternative approach with area-average phase speeds rather than the conventional path-average. Such an approach allows us to incorporate the finite-frequency effects of wave propagation as well as off-great-circle propagation in tomographic inversion. In this chapter, we reformulate the process of surface wave tomography, especially at the regional scale, into a three-stage process working with multi-mode dispersion. The stages consist of the extraction of path-specific information by waveform fitting, construction of multi-mode phase-speed maps as a function of frequency and then a final inversion for local shear wavespeed properties.

Such an approach has been exploited in earlier studies (e.g., Nataf et al., 1986) based on observations of fundamental mode surface waves, however, our new scheme offers the advantage of allowing the incorporation of various styles of information such as multimode dispersion, off-great-circle propagation, and finite-frequency effects within a single formulation. By working directly with phase speed, we can incorporate the deviation of paths from the great-circle using ray-tracing for individual modes and take account of the extended influence zone around each ray path. This approach can be applied not only for regional studies but also for global studies, and, therefore, will be useful for reconciling surface wave tomography at different scales.

5.2 Path-average approximations

The path-average approximation which is now widely adopted in most surface wave studies is based on the analysis of Woodhouse (1974) for surface wave propagation in a stratified medium with slowly varying seismic properties. The propagation of an individual surface-wave mode can be described by an asymptotic ray theory with a high-frequency approximation, and the trajectory is controlled by the phase-speed variations in the model. The local phase speed for the mode is determined by the dispersion characteristics of the local stratified structure in the column beneath the point of interest. The total phase along the path is given by the integral of the local wavenumber k. For the *j*th mode at angular frequency ω , the phase of surface waves, ψ_j , can be represented as,

$$\psi_j(\omega) = \omega \int_{ray_j} \frac{1}{c_j(s,\omega)} ds = \int_{ray_j} k_j(s,\omega) ds,$$
(5.1)

where c_j is the local phase speed of the *j*th mode and the integration of the local wavenumber $k_j(s, \omega)$ is taken along the ray path for the mode. Considering an average wavenumber $\langle k_j(\omega) \rangle$ along the path with a distance Δ for the *j*-th mode, (5.1) can be rewritten as,

$$\psi_j(\omega) = \langle k_j(\omega) \rangle \Delta. \tag{5.2}$$

When the ray path is apart from the great-circle between source and receiver, $\langle k_j(\omega) \rangle$ will be overestimated, because Δ will be shorter than the true path length. As we have shown in chapter 3 (eq. (3.2)), in the presence of slight lateral heterogeneity, the surface waveforms with the leading-order asymptotic approximation can be expressed as,

$$u(\Delta,\omega) = \sum_{j=0}^{J} \mathcal{R}_j(\Delta,\omega) \exp\left[i\left\{\psi_j^0(\omega) + \delta\psi_j(\omega)\right\}\right] \mathcal{S}_j(\omega),$$
(5.3)

where ψ_j^0 is a phase for a reference medium and $\delta \psi_j$ is the perturbation of phase from a reference medium along a ray for the *j*th mode. $S_j(\omega)$ represents the excitation imposed by the source through terms dependent on the source depth. $\mathcal{R}_j(\Delta, \omega)$ includes the terms dependent on receiver depth, the geometric spreading and attenuation of the surface waves. For a laterally varying medium S_j and \mathcal{R}_j are usually evaluated using the structures appropriate to the source and receiver positions. But, as pointed out by Kennett (1995), these contributions are not localised and include some path dependency.

The phase perturbation $\delta \psi_j(\omega)$ between the actual and a reference model can be expressed as,

$$\delta \psi_j(\omega) = \int_0^\Delta k_j(s,\omega) ds - k_j^0(\omega) \Delta$$

=
$$\int_0^\Delta \delta k_j(s,\omega) ds,$$
 (5.4)

where k_j^0 is the wavenumber for a reference model. Introducing a path-average perturbation of the wavenumber,

$$\delta\psi_j(\omega) = \langle \delta k_j(\omega) \rangle \Delta. \tag{5.5}$$

With a restriction to just the variations in shear wavespeed $\delta\beta$ from the reference model $\beta^0(z)$ along the path, the average wavenumber perturbation along a path with the first order approximation is expressed as,

$$\langle \delta k_j(\omega) \rangle = \int_0^R \frac{\partial k_j(\omega)}{\partial \beta(z)} \langle \delta \beta(z) \rangle dz, \tag{5.6}$$

where R is the radius of the Earth and $\langle \delta \beta \rangle$ is the average perturbation of shear wavespeed along the path which can be represented as,

$$\langle \delta\beta(z)\rangle = \frac{1}{\Delta} \int_0^\Delta \left(\beta^{true}(s,z) - \beta^0(z)\right) ds = \frac{1}{\Delta} \int_0^\Delta \delta\beta(s,z) ds.$$
(5.7)

A schematic flow chart of the conventional two-stage inversion scheme based on the exploitation of path-average models is shown in Fig 5.1. In most two-stage methods for 3-D shear wavespeed structure, the stratified model with the path averaged structure derived from waveform fitting in the first stage represents the average of the shear wavespeed (e.g.,



Fig. 5.1. A schematic flow chart of the two-stage inversion for a 3-D shear wavespeed structure using surface waves.

Nolet, 1990) or shear slowness (e.g., Debayle & Kennett, 2000a) structure along the greatcircle path from source to receiver, which is shown as a shaded surface in the bottom panel in Fig 5.1.

5.2.1 Limitations for path-averaged models

The first order perturbation theory for a stratified shear wavespeed model has been the basis of the traditional two-stage approach. This is a reasonable approximation if the variations in seismic structure along the path are sufficiently small and first-order perturbation theory can be applied. However, recent regional tomography models (Simons et al., 1999; Debayle & Kennett, 2000a) indicate that there may be contrasts in crustal and mantle structures, e.g., at the edge of a shield and ocean-continent boundaries, which are strong enough to break down the linearised analysis.

For such models with large velocity variations, we will need to invoke the argument on possible effects from neglecting the higher-order terms in the perturbation theory. Kennett & Yoshizawa (2002) investigated the effects of the second-order term in the shear wavespeed perturbation, and showed that it will become important if there are significant portions of the path with more than about 4% deviation from the path-averaged structure.

If the variations in the true seismic structure along the path are small, it is reasonable to assume that a path-averaged model can be retrieved from a waveform inversion with a stratified model. However, we should note that this is quite a strong requirement; it is not just the variation associated with the path-average model that is required to be small, but also the true deviations from the model. Even in circumstances where the path-average model assumption is inadequate, we can still employ the representation of the seismogram in terms of the integrated phase contributions from each of the modes (5.1), although the paths for the individual modes may be different as seen in Fig 4.12.

In order to investigate the validity of path-average assumption, numerical simulation of surface wave propagation in 3-D models using direct numerical methods or via mode and wavenumber coupling (Kennett, 1998a) will be helpful. However, the numerical implementations for the relatively long paths with respect to wavelengths for simulating regional tomography are still prohibitive. Thus, testing of waveform inversion procedures has been confined to stratified models.

Hiyoshi (2001) studied extensively with synthetic tests for stratified media based on several waveform inversion schemes, and showed that a direct linearised inversion procedure (Nolet et al., 1986) will provide good recovery of the true model for perturbations in velocity of the order of $\pm 2\%$. This implies that the choice of starting model $\beta^0(z)$ is critical to the success of this type of waveform inversion. By employing secondary variables, as in the approach of Cara & Lévêque (1987), larger velocity perturbations up to $\pm 8\%$ from the reference model can be retrieved with inversion for stratified models using Rayleigh waves. Nevertheless, the limitations on the interpretation of the recovered model remain. Only models from the waveform inversion with small variation from the reference model can be regarded as an average of the shear wavespeed structure along the path. For Love waves, the domain of quasi-linearity is rather limited $(\pm 4\%)$, which may be attributed mainly to the significant overlap of the fundamental and higher mode contributions.

The path-average approximation will break down in the presence of rapid changes in seismic parameters compared to the wavelengths of the surface waves. Such strong heterogeneity is likely to produce significant deviations of the surface wave path from the great circle between source and receiver, with induced coupling between modes. As the frequency of the surface waves is increased, the influence of wavespeed gradients become more important. Gradients perpendicular to the propagation path lead to deviations of the propagation path from the great circle as discussed in section 2.7 and gradients along the path tend to cause coupling between modes. These effects limit the frequency ranges over which path average approximations can be applied. The fundamental modes are strongly influenced by shallow structure and suffer from substantial path deviations at higher frequencies. Mode-coupling is more important for the higher modes which sum to represent body-wave contributions (Marquering et al., 1996). The influence of mode coupling can be restricted by choosing a suitable frequency range (Kennett, 1995).

5.2.2 An alternative approach

Instead of simply relying on perturbation theory, we can seek a 1-D model $\beta'(z)$ which gives wavenumbers k'_i for a set of modes,

$$\psi_j(\omega) = k'_j(\omega)\Delta,\tag{5.8}$$

over the frequency range of interest. We can then use $\beta'(z)$ as a representation of the multimode dispersion. This idea arises from the fully non-linear inversion procedure for the waveforms of surface waves developed in chapter 3, based on the use of the Neighbourhood Algorithm procedure of Sambridge (1999) for the exploration of parameter space.

The constraints introduced in waveform inversions for a wavespeed structure are helpful for providing quite stable results of inversion for a particular type of parameterisation for the wavespeed profile. However, different styles of parameterisation produce models with comparable fit to data but different character. Nevertheless, the dispersion characteristics of the different successful models match very well.

If we treat such models recovered from waveform inversion as a summary of multi-mode dispersion, we can work with somewhat less restrictive conditions than working with a path-averaged shear wavespeed model. We still require the Earth to be smoothly varying so that we may employ the path-average approximation for phase, but significant deviations from the reference model can be accommodated. This viewpoint leads naturally to a threestage inversion procedure to recover 3-D seismic structure, through the intermediary of multi-mode dispersion maps as a function of frequency. It is still necessary to work with a limited frequency band so that a simple representation can be used for the propagation terms, avoiding too high frequency (higher than 30 mHz) where mode coupling becomes important.

5.3 Three-stage inversion scheme

We propose a three-stage approach to the construction of 3-D shear wavespeed models from surface wave observations based on the development of multi-mode dispersion maps as a function of frequency (Fig 5.2). This new style of surface wave tomography has the advantage of being able to incorporate information from a wide range of sources in a common framework.

5.3.1 The first stage: Multi-mode dispersion measurements

The first step in the construction of the model is to gather path-specific dispersion information for a number of modes crossing the region of interest. We will require a dense and uniform path coverage to be able to achieve good lateral resolution in dispersion maps at the second step.

Any method to estimate phase dispersion may be used so that the possible maximum ray coverage can be exploited. For long paths to global stations, it may be appropriate to estimate dispersion directly, as used in most global studies (e.g., Trampert & Woodhouse, 1995; Ekström et al., 1997). Such measurements can be extended to higher modes as in the mode stripping technique of van Heijst & Woodhouse (1997), which requires reasonable temporal separation between modal contributions.

Although it would be desirable to use direct extraction of phase speed for different modes, at regional distances, the differences in group velocity for different modes are not sufficient to be able to isolate the contribution of each mode except for the fundamental mode. For Love wave, it is even difficult to separate out the fundamental mode (see Fig 3.8). We therefore need to employ indirect measurements of the phase properties. For regional ranges, we can use waveform inversion for the surface waves as a means of extracting summary 1-D velocity profiles for each path. Irrespective of the particular inversion scheme which has been used for the construction of the path-specific shear wavespeed profile, we can utilise the 1-D model as a representation of the phase dispersion of the surface waves along the path as we have investigated in chapter 3.

Since the 1-D models are just used as a summary of multi-mode dispersion behaviour



Fig. 5.2. A schematic flow chart of the three-stage inversion for a 3-D shear wavespeed model.
along the path, we are able to use isotropic models to treat the dispersion of Love and Rayleigh waves independently. The simplification of model descriptions has a number of significant advantages in increasing the flexibility of the tomographic process.

Previous studies on polarization anisotropy based on the analysis for Love and Rayleigh waves have employed the simultaneous inversion of the waveforms on the vertical and transverse components via a transversely isotropic model with a vertical symmetry axis (e.g., Lévêque et al, 1998; Debayle & Kennett, 2000b). The requirement for good recordings on both components is very restrictive, since we have to avoid the nodal directions of radiation pattern for both Love and Rayleigh waves. By working with just the phase dispersion information for Love and Rayleigh waves represented by independent stratified models that are equivalent to distinct SH and SV wavespeed structures, the condition for simultaneously good recordings for both wave types is not required. We are thus able to exploit all paths for which good recordings are available for either type of waves, which increase the available path coverage.

Another class of useful additional information to the dispersion is the polarization information, which is measured as arrival-angle anomalies of surface waves in the horizontal components. Such information from polarization anomalies can be helpful for both studies of lateral heterogeneity and anisotropy (Laske & Masters, 1996,1998; Larson et al., 1998; Yoshizawa et al., 1999). At regional distances, both Love and Rayleigh waves tend to arrive nearly at the same time, which makes the polarization analysis of surface waves difficult. For longer paths ($\geq 50^{\circ}$), the separation of the different components and modes is clearer and thus we can extract polarization information for the fundamental mode from three-component seismograms.

5.3.2 The second stage: Inversion for multi-mode phase speed maps

The second step of the three-stage inversion is to construct phase dispersion maps as a function of frequency for the different modes using the dispersion information assembled for a number of paths. In this process, the path-average property of the phase along each path is exploited. The advantage of working with phase speeds is that we can readily incorporate off-great-circle propagation and the finite-frequency effects depending on the mode and frequency in the construction of phase speed maps. The dispersion maps can be iteratively updated incorporating with ray tracing taking account of the effects of surface wave propagation.

A first approximation can be carried out as a linear inversion based on the traditional assumption that the surface wave paths follow the great-circle. Each of the dispersion curves for a path can be regarded as a set of linear averaged constraints on the phase speed



100°E 110°E 120°E 130°E 140°E 150°E 160°E 170°E 180°

Fig. 5.3. Illustrations of ray-tracing for Rayleigh waves through the phase speed distribution derived from the shear wavespeed model of Debayle & Kennett (2000a) for the Australian region. A uniform spray of rays is initiated from a source in New Guinea for both the fundamental and first higher mode at 40 s period and tracked across the phase speed maps for the two modes. At the left the sensitivity of the mode contributions to velocity structure with depth are indicated through the partial derivatives with respect to shear wavespeed. To the right the keys indicate the level of phase speed perturbation from reference speeds of 3.93 km/s for the fundamental mode and 4.87 km/s for the first higher mode.

distribution as a function of frequency. We can employ any types of parameterisation of space such as cells or a continuous representation as in the work of Montagner (1986).

This initial inversion is then followed by iterative updates. We can trace surface wave rays directly in the phase speed structure, so that the effects of the deviations of ray paths from the great-circle can be included in the improvement of the phase speed maps at each frequency.

Strong gradients in phase speed can produce significant deviations of ray paths particularly at higher frequencies. In Fig 5.3, we show the patterns of propagation of Rayleigh waves from a source in New Guinea through phase speed distributions for 40 s waves derived from the 3-D shear wavespeed model of Debayle & Kennett (2000a). We show rays radiated with a uniform distribution from the source in phase speed maps for both the fundamental and first higher modes. The strong gradient in phase speed associated with the edge of the Australian shield near 140°E has the effect of introducing defocussing of the rays travelling close to north-south, and the gradients to the east also affect the rays significantly. The effects are more severe for the fundamental mode where the sensitivity is greatest for structure in the uppermost part of the mantle. There is a prominent bunching of rays near the ocean-continent boundaries in northwestern Australia. Even for the first-higher mode, which samples the top 300 km of the mantle, there are noticeable deviations from the great-circle around the edge of the shield and in the Tasman Sea to the east. Two-point ray tracing shows that the deviations from the geodesic path rarely exceed 300 km. However, the focussing and defocussing effects seen in Fig 5.3 should cause some effects on amplitude of observed surface waves, which may result in failure of waveform fitting.

In the phase speed structure, not only ray deviation but also the influence zone surrounding a surface wavepath can be treated. The influence zone for surface waves described in chapter 4 is defined as approximately one-third of the first Fresnel zone, and arises from the finite frequency of the surface waves. This influence zone has a typical half-width, transverse to the path, of about 100 km for the fundamental mode at 40 s period, and increases to around 200 km at 100 s period (Fig 4.11). Introducing the influence zone in a tomographic inversion will naturally smooth out the short-wavelength features from the phase speed maps and corresponds to the effect of the wavefront healing by diffraction processes.

If we can achieve a dense path coverage with a number of crossing paths, we can also constrain the angular variations in phase speed associated with azimuthal anisotropy. Once we have an anisotropic model, anisotropic ray tracing (e.g., Tanimoto, 1987; Larson et al., 1998) should be used to update the dispersion maps.

The use of phase-speed maps at a number of frequencies thus provides a way in which a variety of information can be brought together for mutual benefit. We can use longwavelength phase speed maps derived from global studies as an initial reference model on which the more detailed information from regional paths can be superimposed. This

5.3



the phase speeds 4.01 km/s at 73.7 s and 4.26 km/s at 143.3 s period reference velocity for the wavespeed variation is 4.40 km/s at 100 km and 4.43 km/s at 200 km, and for depth are indicated through the partial derivatives sensitivity peaks at the same depths. Australian region with the phase speed maps for fundamental mode Rayleigh waves at periods where the Fig. 5.4. Comparison of map views of the shear wavespeed model of Debayle & Kennett (2000a) for the The sensitivity of the mode contributions to velocity structure with with respect to shear wavespeed at the right. The

also has the merit of including information from very long-period waves that are not well recorded by portable broad-band instruments that are widely used in regional studies.

There is a close relation between the phase speed variations and the associated 3-D variations in wavespeed as illustrated in Fig 5.4. The phase speed maps reflect the shear wavespeed information through a set of sensitivity kernels derived from the modal eigenfunctions. In Fig 5.4, we displayed tomography maps at depths of 100 km and 200 km of the 3-D model of Debayle & Kennett (2000a) compared with the phase speed distribution for the fundamental mode at frequencies chosen to have the maximum sensitivity at the same depths. These relations can be exploited in the final stage of the three-stage process for a 3-D structure.

5.3.3 The third stage: Inversion for shear wavespeed structure

The final step of the three-stage inversion scheme is to invert local dispersion information for a 1-D shear wavespeed profile. We first need to assemble the full set of multi-mode phase dispersion maps as a function of frequency, and then use some form of cellular inversion to extract a 3-D model. Within each cell or at each grid, we combine the local information for each mode to construct a set of dispersion curves as a function of frequency including azimuthal effects whenever available, and then perform an inversion for a local stratified 1-D wavespeed profile including anisotropy.

The smoothing applied in the construction of the phase-speed maps, both to stabilise the inversion and also through the inclusion of the influence zone at finite frequency, will provide a high degree of correlation between the dispersion properties in nearby cells, and hence in the final shear wavespeed profiles.

We have described the three-stage inversion scheme with the use of the phase dispersion of surface waves, particularly through the influence on seismic waveforms. However, group speed information can also be incorporated in this approach (e.g., Ritzwoller & Levshin, 1998) to better constrain the final 3-D shear wavespeed model. Thus, we can envisage the third stage in the tomographic inversion by undertaking a simultaneous inversion of phase and group speed information for the localised cells (Villaseñor et al., 2001).

At this stage, care must be taken concerning the local sensitivity of short period phase and group speeds to crustal structure. This can be tackled by working with an appropriate 3-D crustal model that represents the shallow structures in the region of interest.

5.4 Discussion

Although the 3-stage tomographic inversion procedure is less direct than conventional 2stage methods such as partitioned waveform inversion, it provides a convenient means of studying regions with large velocity variations in structure. By using the phase speed maps as a function of frequency, we can treat the finite frequency effects of surface wave propagation as well as the influence of strong heterogeneity via ray-path deviation in an iterative linearised inversion. It is also possible to include measurements of the arrival angle (or polarization) anomalies of fundamental-mode surface waves for comparatively longer paths (longer than 50°).

The three-stage inversion allows us to construct the structure utilising information from global studies as well as the information of regional phase dispersion obtained by any convenient means: such as direct measurements or estimates of dispersion derived from 1-D models obtained by waveform inversion. Since the surface wave dispersion does not depend on the particular parametrisation used in the waveform inversion, 1-D models derived from different styles of waveform matching can be combined through their dispersion characteristics.

It is also possible to use different isotropic models for waveform inversion of Love and Rayleigh wave dispersion. This enables us to explore the maximum coverage of Love and Rayleigh wave paths and offers the possibilities of better resolution of anisotropic models. The information from not only the phase dispersion, but also the group dispersion can also be incorporated in the framework of the three-stage approach.

It should be noted that the non-uniqueness in the 1-D models discussed in chapter 3 and section 5.2.2 cannot be eliminated in linearised inversions for 1-D models in the third stage, even though such non-uniqueness can be somewhat suppressed through an appropriate choice of *a priori* information introduced in inversion processes. In this regard, both the two-stage and three-stage scheme may have similar ambiguity in the models, but the three-stage inversion scheme still has advantages over the conventional two-stage approach. That is, various types of information, such as multi-mode dispersion, off-great-circle propagation and finite-frequency effects can be treated efficiently in a common framework, which makes it possible to make more sophisticated treatment when we construct phase speed maps. Furthermore, we can use any convenient technique in the first stage to make reliable measurements of multi-mode phase speeds, and thus different data sets can be combined in tomography models. This scheme is applied to the Australian region in the next chapter.

6.1 Introduction

6

The three-stage inversion scheme introduced in chapter 5 is applied to the Australian region. This approach gives us a number of benefits in that we can treat various sources of information, such as multi-mode dispersion, off-great-circle propagation and finite-frequency effects in a common framework. Therefore, there are significant advantages over conventional two-stage inversion techniques.

The three-stage inversion is a flexible method and any convenient inversion technique can be adopted at each stage. We first estimate multi-mode phase speeds using path-specific 1-D profiles for the Australian paths of Debayle & Kennett (2002), which are derived by fitting the cross-correlograms as secondary observables. The ensemble of the dispersion information is then inverted for multi-mode phase speed maps as a function of frequency using the LSQR algorithm of Paige & Saunders (1982). These phase speed maps are iteratively updated incorporating ray tracing in phase speed maps and the influence zone around surface wave paths. Thus, we obtain several sets of models with different types of inversion (i.e., with or without the effects of the off-great-circle propagation and the finitefrequency) at this stage. Finally, local multi-mode dispersion curves are assembled from the phase speed maps in particular cells, and then, are inverted for local shear wavespeed profiles, which form the final 3-D shear wavespeed model.

Since we obtain several sets of models in the second stage with or without the consideration of various effects of surface wave propagation, several types of 3-D shear wavespeed models with different processes of inversion can be retrieved. In this chapter, we will show five sets of tomography models. With these different types of models, we can investigate the effects of finite-frequency as well as off-great-circle propagation on the final 3-D models. The major objective of this chapter is to present practical formulations for the three independent stages, and to apply them to extract a new Australian upper mantle model to assure the utility of the three-stage approach. Thus, in our first attempt of applying the three-stage approach to surface wave tomography, we simply use a set of Rayleigh-wave phase speeds for the fundamental and the first three modes at particular frequency ranges to obtain an isotropic shear wavespeed model. No effect of anisotropy is considered in the present work. This topic is left to future studies on the improvement of the three-stage method.

6.2 Data set

The first step of the three-stage inversion is to measure multi-mode dispersion from observations. At this stage, we can employ any convenient method for estimating surface wave dispersion. In this study, we use 2000 path-specific 1-D shear wavespeed profiles of Debayle & Kennett (2002), which have been derived from waveform inversion for frequency range between 50 and 160 seconds using secondary observables based on the cross-correlograms calculated from observed and synthetic seismograms (Cara & Lévêque, 1987).

The waveform inversion technique for these 1-D models is the same as that used in an earlier study (Debayle & Kennett, 2000a) for the Australian region, although, in that work, the waveform fitting was performed including slightly shorter periods (down to 40 s of the centre frequency of band-pass filter). Such shorter period surface waves can be contaminated by scattered waves in the crust and the uppermost mantle where strong lateral heterogeneity is expected to exist. Also, in the earlier study, paths from the Philippine Sea region have been included in the data sets. The new data set of Debayle & Kennett (2002) has employed only the paths within the Australian Plate to avoid possible complex effects from major structural boundaries in the north. Therefore, this data set is more desirable to investigate the upper mantle structure beneath the Australian continent.

These 1-D models are derived from a vertical component of Rayleigh waves recorded at the IRIS and GEOSCOPE stations as well as at portable broadband seismic stations of the SKIPPY and KIMBA experiments undertaken by the seismology group at the Australian National University from 1993 to 1998 (Fig 6.1 a). We have corrected the crustal structure using 3SMAC (Nataf & Ricard, 1996) to improve the calculation of shorter period phase speeds. Then we have estimated phase speeds for up to third higher modes for the period ranges shown in Table 6.1 using these path-specific 1-D profiles. With the appropriate corrections for the crustal structure, we can estimate the multi-mode phase speeds to slightly shorter periods of 40 s. The longest period of Rayleigh waves for phase speed



Fig. 6.1. (a) The distribution of the global permanent stations (red triangles), portable stations (green diamonds) and events (violet circles), and (b) 2000 Rayleigh wave paths of Debayle & Kennett (2002).

measurements depends on the modes of interest as given in Table 6.1. The phase speed models are obtained between these period ranges with an increment of 10 s period.

Some examples of 1-D models and estimated phase dispersion curves of the fundamental

mode branch	0	1	2	3
min. period (s)	40	40	40	40
max. period (s)	150	140	100	60

Table 6.1. The minimum and maximum period range for which the Rayleigh-wave phase speeds are estimated from the 1-D shear wavespeed profiles.



Fig. 6.2. Examples of the path-average 1-D profiles of Debayle & Kennett (2002) and multi-mode phase dispersion curves estimated from these 1-D models. The three paths are chosen so that they sample mainly (a) oceanic region, (b) continental region and (c) both oceanic and continental region.

and the first three modes for different paths to the NWAO station in the south-west Australia are shown in Fig 6.2. These 1-D models are smoothed at 400 and 670 km boundaries. A path passing through the Indian Ocean (Fig 6.2 a), shows clear slower shear wavespeed anomaly around the 150 km depth, whereas a continental path (Fig 6.2 b) shows the noticeable higher wavespeed anomalies in the top 200 km, passing mainly in the Proterozoic and Archaean blocks in the central and western Australia (Fig 6.3). Another example in Fig 6.2 (c) shows a path traveling both the oceanic and continental region. The corresponding path-average 1-D model shows the average features of the oceanic and continental structures and there is no remarkable anomalies in the upper mantle.



Fig. 6.3. Major geological blocks at the surface of the Australian region.

The use of the sets of 1-D models of Debayle & Kennett (2002) is helpful to assess the final model from the three-stage approach compared with a model obtained from a different technique, i.e., two-stage approach. The inversion procedure adopted in Debayle & Kennett (2000a, b and 2002) is based on the two-stage approach working with pathaverage 1-D models which are used to constrain the final 3-D model. In this study, we will also obtain a 3-D shear wavespeed model with two-stage procedure using such 1-D models directly.

In practice, the available number of higher modes, which can be reliably extracted from the observation, depends strongly on the excitation of the modes at the source. Some of these 1-D models are primary constrained by the fundamental mode for shallow events. For testing the development of the method, we have used the 1-D models to generate higher mode dispersion, even where the model is almost entirely constrained by the fundamental mode. Thus for very shallow sources, the higher-mode dispersion will not represent independent information.

The reliability of the measured phase speeds from such 1-D models can be taken into account by using *a posteriori* errors in the 1-D shear wavespeed profiles, which represent how well the model is constrained via waveform fitting; i.e., for 1-D models that have been constrained only by the fundamental mode, the estimated errors in shear wavespeed become large at depth where there is little sensitivity. We use this information for measuring

the errors in the estimated phase speeds, which are subsequently used as weighting on the phase speed data in inversions for phase speed maps at the second stage.

6.3 Inversion for multi-mode phase speed maps

In this section, we first explicitly formulate the phase speed inversion for the secondstage. Any method for linearised inversions can be used as long as they are able to give reliable and stable solutions for intermediate size of tomographic system. In this study, we have employed the LSQR algorithm (Paige & Sanders, 1982) to invert the sets of path-specific dispersion data for multi-mode dispersion maps as a function of frequency. The formulation of the inversion and model resolution, as well as the trade-off between the misfit and model norm are explained in detail. Some examples of phase speed maps derived from different types of inversion kernels are also displayed.

6.3.1 Formulation of inversion

In the second step of the three-stage inversion, the ensembles of the path-average phase speeds for each mode are inverted for mode-dependent phase speed maps as a function of frequency. The linear relationship between perturbation of phase of seismograms, $\delta\psi$, and phase speeds for a *j*th mode, δc^{j} , can be represented as (e.g., Woodhouse and Wong, 1986),

$$\delta\psi_j(\omega) \simeq -k_j(\omega) \int_{ray_j} ds \frac{\delta c^j(s,\omega)}{c_0^j(\omega)},\tag{6.1}$$

where k_j is the wavenumber $(= \omega/c_0^j)$ for a reference model. Hereafter we omit the dependency on a frequency ω and mode number j. The phase of observed seismogram is thus represented by the average phase speed perturbation along the great-circle with an epicentral distance Δ ,

$$\delta \psi^{obs} \simeq -k \left\langle \frac{\delta c}{c_0} \right\rangle^{obs} \Delta,$$
(6.2)

where

$$\left\langle \frac{\delta c}{c_0} \right\rangle^{obs} = \frac{\langle c \rangle^{obs} - c_0}{c_0},\tag{6.3}$$

and $\langle c \rangle^{obs}$ is a average phase speed along the great-circle. In the process of waveform inversion, synthetic seismograms are generally calculated for propagation along the great circle between the source and receiver, and thus the estimated phase speeds are also regarded as being measured as an average along the great-circle rather than along the actual ray path. When we update phase speed maps working with ray tracing, we need to



Fig. 6.4. Illustration of the great-circle with epicentral distance Δ and the actual ray path. Since the phase of an observed seismogram is constant, the average phase speed $\langle c \rangle^{obs}$ measured along the great-circle and $\langle c \rangle^{ray}$ along the ray have slight differences associated with the differences in the travel distances.

correct the observed phase speeds along the great-circle so that they are to be measured along the ray path, as discussed in the next section.

6.3.1.1 Linear relation

The path-average phase speeds can be given as the following linear relationship derived from (6.1) and (6.2),

$$\left\langle \frac{\delta c}{c_0} \right\rangle^{obs} = \frac{1}{\Delta} \int_{g.c.} ds \frac{\delta c(s)}{c_0},\tag{6.4}$$

where the integration is taken along the great-circle between the source and receiver. Using the relationship (6.4), frequency and mode dependent phase speed maps are obtained based on the assumption of surface wave propagation along the great-circle.

In the framework of the three-stage inversion introduced in the previous chapter, we can update such phase speed maps by working with ray tracing for all the paths. Since the actual ray paths should be longer than the epicentral distance Δ along the great-circle path, and the phase contribution in a seismogram does not depend on the ray path (Fig 6.4), the average phase speeds $\langle c \rangle^{obs}$ along the great-circle can be corrected as follows. The observed phase $\psi(\omega)$ can be simply regarded as an integral of phase slowness along an arbitrary path,

$$\psi(\omega) = \frac{\omega}{\langle c \rangle^{obs}} \Delta = \omega \int_{ray} \frac{1}{c(s)} ds = \frac{\omega}{\langle c \rangle^{ray}} \Delta_{ray}.$$
(6.5)



Fig. 6.5. Schematic illustration of the influence zone in a ray-centered coordinate system (s, n).

Thus the average phase speed $\langle c \rangle^{ray}$ along the actual ray path with a distance, $\Delta_{ray} = \int_{ray} ds$, can be represented as,

$$\langle c \rangle^{ray} = \langle c \rangle^{obs} \frac{\Delta_{ray}}{\Delta} \tag{6.6}$$

Using (6.6), the linear relation for the average phase speed along the ray can be written as,

$$\left\langle \frac{\delta c}{c_0} \right\rangle_{ray} = \frac{\langle c \rangle^{ray} - c_0}{c_0} = \frac{1}{\Delta_{ray}} \int_{ray} ds \frac{\delta c(s)}{c_0}.$$
(6.7)

Phase speed maps can be improved not only via ray tracing, but also by including the influence zone of surface wave paths explained in chapter 4. Since the influence zone has been defined as the finite area over which surface waves are coherent in phase, we can regard the observed phase speeds as an average within the influence zone rather than just as an average along the path.

Using the ray centered coordinate system (s, n) introduced in chapter 4, we first consider the average phase speed variation perpendicular to the ray at a particular point s on the path,

$$\left\langle \frac{\delta c(s)}{c_0} \right\rangle = \frac{1}{2\mathcal{N}(s)} \int_{width} dn \frac{\delta c(s,n)}{c_0},\tag{6.8}$$

where $\mathcal{N}(s) = \int_0^{\mathcal{N}(s)} dn$ is a half-width of the influence zone at a point s on the ray path (Fig 6.5). (6.8) can then be integrated along the path to give an average phase speed variations within the influence zone,

$$\left\langle \frac{\delta c}{c_0} \right\rangle = \frac{1}{\Delta} \int_{path} ds \left\langle \frac{\delta c(s)}{c_0} \right\rangle$$

$$= \frac{1}{\Delta} \int_{path} ds \frac{1}{2\mathcal{N}(s)} \int_{width} dn \frac{\delta c(s,n)}{c_0}.$$
(6.9)

The integration term along a path can be calculated along either the great-circle path or the actual ray path. If we take into account the influence zone around the source and receiver locations as discussed in the Appendix B, the integration along the path should be undertaken between the two edges of the influence zone on the ray trajectory (Fig 6.5), so that the total zone should be slightly longer ($\sim \lambda/2$) than the ray path length. Since the influence zone is defined so that the surface waves within a zone is coherent in phase, we can regard the measured phase speed variation along a path as the average within the influence zone.

It should be noted that we do not simply take an area-average within the influence zone in (6.9). The simplest representation of the area-average can be given by dividing the integration of phase speeds within the influence zone by the total area of the zone $\mathcal{A}(=\int_{path} ds \int_{width} dn)$. But, in (6.9), we first consider the integral perpendicular to the ray, then, the average phase speeds over the width of the influence zone are further integrated along the path. This process of double integration allows us to provide a two-dimensional distribution of the sensitivity to the phase speed structure, which varies along the path but is constant over the width of the influence zone as visualised in Fig 6.6. From (6.2) and (6.9), we can obtain a form of sensitivity kernels of surface wave phase as,

$$\delta \psi = -\frac{\omega}{c_0} \int_{path} ds \frac{1}{2\mathcal{N}(s)} \int_{width} dn \frac{\delta c(s,n)}{c_0} \\ = -\int_{path} ds \int_{width} dn \, \mathcal{K}_{\psi}(s,n) \, \frac{\delta c(s,n)}{c_0},$$
(6.10)

where the sensitivity kernel \mathcal{K}_{ψ} is given by,

$$\mathcal{K}_{\psi}(s,n) = \begin{cases} \frac{\omega}{2\mathcal{N}(s)c} & : |n| \le \mathcal{N}(s) \\ 0 & : |n| > \mathcal{N}(s) \end{cases}$$
(6.11)

At the extended locations over the source and receiver derived from the edge corrections in Appendix B, the half-width $\mathcal{N}(s)$ is fixed to be $\lambda/2$ in the practical calculation of the kernel (6.11) so that we can avoid a very large a sensitivity caused by too small half-width of the influence zone.

As seen in Fig 6.6, it is apparent that the surface wave sensitivities to the phase speed structure varies along the path, whereas they are constant over the width of the influence zone. The highest sensitivity is concentrated near the source and receiver, as expected from the sensitivity kernels evaluated with first-order scattering theory (e.g., Yomogida, 1992; Marquering et al., 1998, 1999; Dahlen et al., 2000; see also chapter 7).

Although the sensitivity kernels \mathcal{K}_{ψ} do not vary across the central ray path within the influence zone, the assumption of the coherent phases within the zone tends to be violated near the edge of the influence zone as discussed in section 4.4.3. The errors, caused by the



Fig. 6.6. Representation of the sensitivity kernel \mathcal{K}_{ψ} for a path between an event in Fiji and the CAN station in south-eastern Australia. Influence zone of the fundamental-mode Rayleigh waves at 50 (a and c) and 100 (b and d) seconds are displayed. In (c) and (d), a cosine-taper weight function $\mathcal{W}(s,n)$ is multiplied to the \mathcal{K}_{ψ} . PREM model is used as the reference model used to calculate all the kernels. The yellow line is the great-circle.

slight incoherency in phase, can be somewhat reduced by introducing a weight function across the influence zone. Considering the very small variations in the phase contributions within the influence zone, we adopt a cosine taper, rather than a Gaussian, as the weight function. The equation (6.9) is then represented as,

$$\left\langle \frac{\delta c}{c_0} \right\rangle = \frac{1}{\Delta} \int_{path} ds \frac{1}{2\mathcal{N}(s)} \int_{width} dn \mathcal{W}(s,n) \frac{\delta c(s,n)}{c_0},\tag{6.12}$$

where

$$\mathcal{W}(s,n) = \cos\left[\frac{\pi}{2} \left(\frac{n}{\mathcal{N}(s)}\right)^2\right].$$
(6.13)

In the actual inversion for the phase speed maps, we use the formulation (6.12) with a weight function.

To take account of the full effects of scattering outside the influence zone, we may envisage the use of the Born approximation with more rigorous calculations of the sensitivity kernels. Tackling these more complicated problems is beyond the scope of this study, but such rigorous sensitivity kernels will be discussed in chapter 7.

6.3.1.2 Least-squares inversion

The linearised equations (6.4), (6.7) and (6.12) can be written in a generalised form,

$$\mathbf{d} = \mathbf{G}\mathbf{m},\tag{6.14}$$

where the data vector **d** consists of the observed phase speed variations $\langle \delta c/c \rangle_i (i = 1, 2, ..., M)$ and M is the total number of paths. **m** is a model vector that include the model parameters $m_j (j = 1, 2, ..., N)$ and **G** is the kernel matrix.

In this study, we use a spherical B-spline function $\mathcal{F}(\theta, \phi)$ (e.g., Lancaster & Salkauskas, 1986) defined at the centre of a geographic cell as a basis function to expand the phase speed perturbation as follows,

$$\frac{\delta c(\theta,\phi)}{c_0} = \sum_{j=1}^N m_j \mathcal{F}_j(\theta,\phi),\tag{6.15}$$

where m_j is the coefficients of the *j*th basis function \mathcal{F}_j , which are the model parameters to be obtained, and N is the total number of the model parameters. The spatial parameterisation using such basis function is explained in detail in Appendix C.

Using the spherical B-splines, the components of the kernel matrix \mathbf{G} can be written as,

$$G_{ij} = \begin{cases} \frac{1}{\Delta_i} \int_0^{\Delta_i} ds \mathcal{F}_j & : \text{ average along the path,} \\ \frac{1}{\Delta_i} \int_0^{\Delta_i} ds \frac{1}{2\mathcal{N}(s)} \int_{width} dn \mathcal{W}(s,n) \mathcal{F}_j & : \text{ average within the influence zone,} \end{cases}$$
(6.16)

for the *i*th path and the *j*th model parameter, where the epicentral distance Δ_i is measured along either the great-circle or the ray path.

We can solve the linearised equation (6.14) with the damped least-squares inversion scheme, minimising the following objective function,

$$\Phi(\mathbf{m}) = (\mathbf{d} - \mathbf{G}\mathbf{m})^T \mathbf{W}_d (\mathbf{d} - \mathbf{G}\mathbf{m}) + \lambda^2 \mathbf{m}^T \mathbf{m},$$
(6.17)

where \mathbf{W}_d is a weighting matrix that controls the relative contribution of individual data misfit, and λ is an arbitrary damping parameter that controls the trade-off between the model variance and resolution, which subsequently affects the maximum amplitude and smoothness of the model.

In this study, we use a data covariance matrix \mathbf{C}_d to represent the weighting on the data misfit, so that $\mathbf{W}_d = \mathbf{C}_d^{-1}$. Assuming that measured phase speeds for different paths are uncorrelated (although there may be some correlation through the source location) and their variances are different for each path, the covariance matrix can be represented as,

$$\mathbf{C}_d = \sigma_{d_i}^2 \mathbf{I},\tag{6.18}$$

where σ_{d_i} is the measurement errors for the *i*th datum and **I** is the identity matrix. Then, the first term in the right-hand side of (6.17) can be represented explicitly as,

$$\sum_{i} \frac{1}{\sigma_{d_i}^2} \left| d_i - \sum_{j} G_{ij} m_j \right|^2 = \sum_{i} \left| \frac{1}{\sigma_{d_i}} d_i - \sum_{j} \frac{1}{\sigma_{d_i}} G_{ij} m_j \right|^2 = \sum_{i} |d'_i - \sum_{j} G'_{ij} m_j|^2, \quad (6.19)$$

where

$$d'_{i} = \frac{1}{\sigma_{d_{i}}} d_{i}, \qquad G'_{ij} = \frac{1}{\sigma_{d_{i}}} G_{ij}.$$
 (6.20)

Introducing new data vector \mathbf{d}' and kernel matrix \mathbf{G}' which are scaled with measurement errors for each datum as shown in (6.20), we can rewrite (6.17) as follows,

$$\Phi(\mathbf{m}) = |\mathbf{d}' - \mathbf{G}'\mathbf{m}|^2 + \lambda^2 |\mathbf{m}|^2.$$
(6.21)

Thus our inverse problem is to solve the following system of equation,

$$\begin{bmatrix} \mathbf{G}'\\\lambda\mathbf{I} \end{bmatrix} \mathbf{m} = \begin{bmatrix} \mathbf{d}'\\\mathbf{0} \end{bmatrix}.$$
(6.22)

We use the LSQR algorithm (Paige & Saunders, 1982) to solve a set of linear equations (6.22).

6.3.2 Five sets of phase speed models

In this study, we obtain multi-mode phase speed models using different types of inversion kernels (6.16) working with the great-circle approximation, ray tracing or the influence

Model Relation Chart



Initial models

GC0 : models with the great-circle approximationGCiz : models with the influence zone around the great-circle path

Updated models

Ray-GC0	: models updated from GC0 with ray tracing only
Riz-GC0	: models updated from GC0 with ray tracing and
	the influence zone
Riz-GCiz	: models updated from GCiz with ray tracing and
	the influence zone

Fig. 6.7. A chart representing relations amongst five sets of models. Model sets GC0 and GCiz are derived from the great-circle approximation, and these models are used as initial heterogeneous models in the subsequent inversions for updating models (*Ray-GC0*, *Riz-GC0* and *Riz-GCiz*).

zone. In order to distinguish the different sets of phase speed models derived from different techniques, they are named according to their inversion method. The naming convention and the relation of these model-sets are summarised in a model chart (Fig 6.7).

We first obtain the set of models $GC\theta$ using the great-circle approximation without any consideration of the zone of influence. Therefore, the model set of $GC\theta$ is based on a similar procedure to the conventional inversion technique for phase speed maps.

The influence zone can be incorporated with the great-circle approximation. We generate a set of models *GCiz* using the influence zone around the great-circle paths. Since we assume that each path is along the corresponding great-circle, we simply require a homogeneous reference model. We use the Preliminary Reference Earth Model (Dziewonski & Anderson, 1981) as a reference model to represent all the tomography models shown in this chapter.

The phase speed models $GC\theta$ can be updated by incorporating surface-wave ray tracing in the phase speed maps, which produce a new set of models Ray- $GC\theta$. In the sense of geometrical ray theory, these models updated with ray tracing can be considered to be a better set of models than GC0.

We also use $GC\theta$ to obtain an alternative new model set Riz- $GC\theta$ considering both the off-great-circle propagation and the influence zone for each path.

The other set of the heterogeneous models GCiz derived from considering the influence zone around the great-circle, are further updated working with ray tracing as well as with the influence zone to produce the final model set *Riz-GCiz*, which can be regarded as potentially the best model in this study in that we take full account of the possible effects of off-great-circle propagation and finite-frequency effects.

We term these update processes for the sets of models as *global iteration* to distinguish them from the other iteration processes for each phase speed inversion with the LSQR algorithm (*local iteration*).

Although the new sets of phase speed models (i.e., Ray-GC0, Riz-GC0 and Riz-GCiz) can be further updated by iteration, there is no need to repeat this process more than once because, in most cases, the second global iteration do not alter (or improve) the models significantly. As shown in the later sections, the models Riz-GC0 and Riz-GCiz, which are updated from a different initial models by considering both the off-great-circle paths and finite-frequency effects, are extremely similar suggesting that one global iteration is sufficient to provide a satisfactory convergence of phase speed models including the effects of finite frequency.

Still, we should note that each phase speed model is obtained through more than 20 local iterations using the LSQR algorithm as explained in the next section.

6.3.3 Model assessment: trade off and resolution of models

In this section we discuss the behaviour of the tradeoffs between data misfit and model norm, and a way to assess the spatial resolution of the models.

Fig 6.8 (a) shows the behaviours of the misfit and model norms as a function of local iterations of the LSQR algorithm, with varying damping parameters ($\lambda = 0.4$, 1.0 and 1.6) for a phase speed model $GC\theta$ at 100 s. The number of local iteration that give a sufficient convergence depends on the choice of damping parameter. For a relatively large damping of $\lambda = 1.6$, both the model and misfit norms converge by the 10th iteration with a suppressed model norm and slightly higher misfit. With a smaller damping of $\lambda = 0.4$, the model norm grows rapidly and models do not converge within 10 iterations.

With $\lambda = 1.0$, the model reaches a satisfactory level of convergence within 10 iterations with a compromise in the trade-off between the misfit and the model norm. The tradeoff curve is displayed in Fig 6.8 (b), in which the misfit is represented by [100 - variance



Fig. 6.8. (a) The model and misfit norms as a function of local iterations of the LSQR algorithm for a phase speed model GC0 at 100 s. (b) Trade-off curves for varying damping parameter λ . We choose $\lambda = 1.0$ as a preferred damping for this example.

40

50

60



(a) variance reduction for phase speed maps at 60 s

Fig. 6.9. (a) Variance reductions for five sets of Rayleigh-wave phase speed models at 60 s as a function of mode. (b) Variance reductions for the phase speed models $GC\theta$ for the first four modes as a function of frequency. Other model sets also provide very similar variance reductions to this.

period (s)

90 100 110 120 130 140 150

80

70

reduction(%)], where we define the variance reduction as $(1 - |\mathbf{d} - \mathbf{Gm}|^2 / |\mathbf{d}|^2) \times 100(\%)$. Thus, in this example, we chose $\lambda = 1.0$ as an appropriate damping for this model.

The other phase speed models for different frequencies and different modes show a similar behaviour to Fig 6.8 with slight differences in the values of the damping parameter. For each phase speed map, we choose appropriate damping which give the similar tradeoff behaviour in Fig 6.8. In this study, as a suitable damping with a good compromise for tradeoffs, we use λ from 0.8 to 1.2 for the fundamental modes, and from 0.6 to 0.8 for

the higher modes, depending on the frequency. The trend of tradeoff is also similar in the other four sets of models in Fig 6.7. Therefore, we use the same damping parameter as used in GC0 for corresponding phase speed maps of the other model sets. Although the models usually converge within 20 iterations with appropriate damping parameter λ , we calculate up to 50 iterations to ensure a smaller misfit.

We show the variance reductions (VR) calculated for the five sets of phase speed models of all modes at 60 s in Fig 6.9 (a). There are almost no conspicuous differences in the VR between different sets of models, although the models derived using the influence zone (i.e., GCiz, Riz-GC0 and Riz-GCiz) show subtly smaller values of VR.

In Fig 6.9 (b), the variance reductions for all phase speed models of GC0 are displayed as a function of frequency. The fundamental mode models tend to show somewhat smaller VR of around 60-70 %, whereas the higher-mode models achieve higher VR of around 80 %. The other sets of models also show similar trend to Fig 6.9 (b).

Unlike the tomographic studies based on simple geometrical ray theory, it is not trivial to visualise the resolution for the final tomography models considering finite frequency effects, since such effects of finite frequency cannot simply be fully reconstructed by a linearised forward modeling (as has been used in most conventional checker-board type tests). One of the most desirable ways to assess our models would be to use numerical waveform modeling in a full 3-D structure for all the source and receiver pairs and then to invert them for a 3-D model using our three-stage approach. However, such numerical techniques are not fully developed, yet require heavy computation and are prohibitive to apply to large-scale inversion.

A simple measure of the resolution of the models can be derived from assessing the diagonal elements of a resolution matrix **R**. The estimated solution of models \mathbf{m}^{est} with the weighted damped least-squares inversion can be given by (e.g., Menke, 1984),

$$\mathbf{m}^{est} = \left[\mathbf{G}^T \mathbf{W}_d \mathbf{G} + \lambda^2 \mathbf{I}\right]^{-1} \mathbf{G}^T \mathbf{W}_d \mathbf{d}^{obs}.$$
(6.23)

Since the data vector is related to the true model vector \mathbf{m}^{true} with the linear relation $\mathbf{d}^{obs} = \mathbf{G}\mathbf{m}^{true}$, we can obtain the following equation for the model resolution matrix in a weighted damped least-squares problem,

$$\mathbf{m}^{est} = \left[\mathbf{G}^T \mathbf{W}_d \mathbf{G} + \lambda^2 \mathbf{I}\right]^{-1} \mathbf{G}^T \mathbf{W}_d \mathbf{G} \mathbf{m}^{true},$$
(6.24)
= $\mathbf{R} \mathbf{m}^{true},$ (6.25)

where $\mathbf{R} = [\mathbf{G}^T \mathbf{W}_d \mathbf{G} + \lambda^2 \mathbf{I}]^{-1} \mathbf{G}^T \mathbf{W}_d \mathbf{G}$ is the model resolution matrix. If \mathbf{R} is the identity matrix for which all the diagonal elements are 1, each model parameter has been determined uniquely. However, in general, \mathbf{R} contains off-diagonal elements with the peak around the diagonal components, and so there are some correlations between the model

parameters. The damping parameters applied in the inversion also suppress the resolution to some extent.

The diagonal elements of **R** can be expanded in the spherical B-spline basis, with an appropriate normalisation n_j for the *j*th basis function at a particular location (θ, ϕ) such that $\sum_j^N n_j \mathcal{F}_j(\theta, \phi) = 1$, to project the spatial resolution into a map. If all the diagonal terms are 1, the resolution maps will be unity everywhere in the region of interests. Several examples of the resolution maps are shown in the subsequent sections together with the corresponding phase speed maps.

6.3.4 Multi-mode phase speed maps for Rayleigh waves

In this section we display several sets of phase speed maps at chosen periods. Extensive sets of multi-mode phase speed maps Riz-GCiz are displayed in Appendix D. The five types of phase speed maps of the fundamental-mode Rayleigh wave at 100 s are displayed in Fig 6.10. As we can expect from the subtle differences in the diagrams of the variance reductions (Fig 6.9), the five sets of models shows some degree of consistency. Thus, all these models share similar features, that is, the fast velocity anomalies are seen in the Archaean and Proterozoic blocks in the central and western Australia, whereas the slow velocity anomalies lie in the eastern Phanerozoic region of the eastern Australia and in the Coral and Tasman Sea (Fig 6.3).

The resolution of these models depends strongly on the coverage and types of the paths. The number of ray paths counted in $2^{\circ} \times 2^{\circ}$ cells for the model sets *GC0* and *Riz-GC0* at 100 s are visualised in Fig 6.11. For the finite-width rays, we count the cells which are covered by the influence zone. Therefore, we can see more uniform coverage in Fig 6.11 (b) due to the finite-width of the rays, whereas the ray coverage with simple geometrical ray is rather spotty. The other models with geometrical ray theory give the similar ray coverage to Fig 6.11 (a) even with or without ray bending, whereas all the models with the influence zone provide the coverage like in Fig 6.11 (b).

The spatial resolution maps, which are estimated from the diagonal elements of the resolution matrix \mathbf{R} , for the phase speed models in Fig 6.10 are displayed in Fig 6.12. Due to the effects of damping applied in the inversions, the maximum resolution is suppressed and cannot reach unity.

The models with geometrical ray theory (Fig 6.12 (a) & (b)) show higher resolution where the coverage of path are sufficient, whilst the finite-frequency models (Fig 6.12 (c), (d) and (e)) show rather lower resolution due to inclusion of the spatial integration around the paths. Although the latter models show lower resolution, the differences in the resolution in the whole region of interest are rather small, that is, we can achieve more



Fig. 6.10. Five types of phase speed maps of the fundamental-mode Rayleigh waves at 100 s. All the models are shown as a perturbation from PREM. (a) A model GC0 derived from the great-circle approximation. (b) An updated model Ray-GC0 with surface-wave ray tracing in the model (a). (c) An updated model Riz-GC0 with ray tracing as well as the influence zone. (d) A model GCiz with the influence zone around the great-circle paths. (e) An updated model Riz-GCiz from (d) using ray tracing and the influence zone.



Fig. 6.11. Density plot of the number of ray paths counted in $2^{\circ} \times 2^{\circ}$ cells for models (a) *GC0* and (b) *Riz-GC0* at Rayleigh wave 100 s. In case of (b), cells which are covered by the influence zone are also counted.

uniform spatial resolution. This is quite helpful for the assessment of the models with unevenly covered models, which is the case for most tomography models at present.

Now let us closely look at the differences in the models shown in Fig 6.10. Some region where there are strong local heterogeneity in the ray theoretical models (GC0 and Ray-GC0) are smoothed out in the finite-frequency models (GCiz, Riz-GC0 and Riz-GCiz). This effect is especially conspicuous in the Tasman Sea and in the Proterozoic blocks in



Fig. 6.12. Map projections of the diagonal elements of the resolution matrix **R** for the models in Fig 6.10. The same damping of $\lambda = 1.0$ is applied to all the models.

the central Australia. We can also see the extremely good spatial correlations between the updated models with finite frequency Riz-GC0 and Riz-GCiz, which gives the correlation coefficients of more than 0.99 whilst the correlations between the other models are just around 0.95.

Other examples of phase speed models of the first three modes for GC0 and Riz-GC0 at 60 s are displayed in Fig 6.13. We now see the effects of the finite frequency on the different modes at the same frequency. The smoothing effects, which arise from the inclusion of the finite-frequency effects in the inversion process, are common in all the updated models, but the effects are clearer in the higher-mode models because of the wider influence zone arising from the faster absolute phase speeds.

The velocity perturbation in the higher mode models are usually smaller than the fundamental mode model implying that the velocity perturbation in the deeper part of the mantle can be comparatively small. Therefore, for the higher mode models, the effects of off-great-circle propagation is not critical, resulting in almost no differences in the configuration of the velocity structure in GC0 and Riz-GC0. However, for the fundamental mode at 60 s, we can see a clear difference in the boundaries between the lower and faster phase speed anomalies in the eastern Australia, which indicates that the importance of the effects of ray-bending in shorter period waves.

The projections of the corresponding resolution maps are shown in Fig 6.14. Again, we see the suppressed local resolution but more uniform resolution in the finite frequency models. Although the ray paths for these models are the same, there are differences in the resolution of models due to the damping parameter and the weighting applied to the data.

6.4 Inversion for local shear wave speed models

The third stage is to invert for a set of local shear wavespeed model using local multi-mode dispersion information assembled from a set of phase speed maps derived in the second stage. In this section, the inversion for the shear wavespeed structure is formulated using the least-squares generalised inversion scheme of Tarantora & Valette (1982).

6.4.1 Formulation of inversion

Since the nonlinear inversion procedure of Tarantora & Valette (1982), which has been widely used in much of surface wave studies (e.g., Montagner & Jobert, 1981; Nataf et al., 1986; Cara & Lévêque, 1987; Nishimura & Forsyth, 1989), has already been explained in many papers, we only briefly explain the method of inversion.



Fig. 6.13. Two sets of phase speed maps of the first three modes of Rayleigh waves at 60 s. All the models are shown as a perturbation from PREM. (a) The fundamental-mode model $GC\theta$ derived from the greatcircle approximation. (b) The fundamental-mode model *Riz-GC0* updated from (a) considering both the influence zone and ray path bending. (c) Same as (a) but the first higher mode model. (d) Same as (b) but the first higher mode model updated from (c) (e) Same as (a) but the second higher mode model. (e) Same as (b) but the first higher mode model updated from (e)



Fig. 6.14. Map projections of the diagonal elements of the resolution matrix **R** for the corresponding models in Fig 6.13. The damping for these models are $\lambda = 1.2$ for (a) and (b), $\lambda = 0.8$ for (c) and (d), and $\lambda = 0.6$ for (e) and (f).

6.4.1.1 Generalised nonlinear inversion

We consider a problem where the observed data \mathbf{d} is represented as a function of model parameters \mathbf{p} as follows,

$$\mathbf{d} = \mathbf{g}(\mathbf{p}),\tag{6.26}$$

where **d**, in this case, consists of a set of local multi-mode phase speed perturbation $\delta c(\omega)$ as function of ω . The model parameter vector **p** consists of a 1-D profile of the local shear wavespeed perturbation $\delta\beta(z)$ as a function of depth z. The phase speed perturbation depends also on the P wave speed and density, but in the intermediate period range, which is of interest in this study, there are only a slight effects from these parameters. Therefore, we only consider the shear wavespeed perturbation in this study.

With an appropriate reference model \mathbf{p}_0 , a model parameter vector at the (k + 1)th iteration for an underdetermined problem can be extracted as (Tarantola & Valette, 1982),

$$\mathbf{p}_{k+1} = \mathbf{p}_0 + \mathbf{C}_{pp} \mathbf{G}_k^T \left(\mathbf{C}_{dd} + \mathbf{G}_k \mathbf{C}_{pp} \mathbf{G}_k^T \right)^{-1} \left[\mathbf{d} - \mathbf{g}(\mathbf{p}_k) + \mathbf{G}_k (\mathbf{p}_k - \mathbf{p}_0) \right],$$
(6.27)

where \mathbf{p}_k is the model vector at the *k*th iteration, \mathbf{C}_{dd} is the *a priori* data covariance matrix, \mathbf{C}_{pp} is the *a priori* model covariance matrix and \mathbf{G}_k the kernel matrix which consists of the partial derivatives of the data with respect to the model parameters with components, for the *i*th data and *j*th model parameter,

$$G_{ij} = \frac{\partial d_i}{\partial p_j}.$$
(6.28)

In this study, (6.28) represents the partial derivatives of phase speeds with respect to shear wave speeds.

The model resolution matrix for the algorithm of Tarantora & Vallette (1982) can be given by (e.g., Montagner & Jobert, 1981),

$$\mathbf{R} = \mathbf{C}_{pp} \mathbf{G}_k^T \left(\mathbf{C}_{dd} + \mathbf{G}_k \mathbf{C}_{pp} \mathbf{G}_k^T \right)^{-1} \mathbf{G}_k.$$
(6.29)

The nature of the resolution matrix \mathbf{R} is the same as that discussed in the previous section for the inversions in the second-stage. By investigating the rows of \mathbf{R} , we can check how well the model parameters has been resolved.

6.4.1.2 A priori information

We assume that the *a priori* model covariance matrix \mathbf{C}_{pp} can be represented by a Gaussian distribution as follows,

$$C_{pp}(r_i, r_j) = \sigma(r_i)\sigma(r_j) \exp\left\{-\frac{1}{2}\frac{(r_i - r_j)^2}{L^2}\right\},$$
(6.30)

where r_i is the depth of the *i*th model parameter, $\sigma(r_i)$ is the standard deviation for the *i*th model parameter and L is the average correlation length between the model parameters r_i and r_j .

The standard deviation σ constrains the amplitude of variations of model parameters, and the correlation length L controls the smoothness of the model variations with depth, and so determines how rapidly the model can vary as a function of depth. The choice of these parameters depends simply on one's preference.

Previous studies have shown that the shear wavespeeds in the upper mantle vary from peak to peak around 0.5 km/s over 100 km (e.g., Nishimura & Forsyth, 1989). We prefer models which vary smoothly with depth but allow some degree of rapid variation so that we can treat quick change with large wavespeed perturbation. Therefore, we choose the value of correlation length $L \approx 20$ km, and the $\sigma \approx 0.1$ km/s over the depth range below Moho. For the shallower structure above the Moho, the correlation length is chosen to be $L \approx 5$ km so that more rapid variation in the shallower structure is allowed.

As reference models used to initiate the inversion, we use the PREM for the oceanic region and the PREMC, whose upper mantle structure is modified to represent the continental structure, for the continental region. Both PREM and PREMC models are modified to have smooth variation across the 220 km, 400 km, and 670 km boundaries. Prior to the inversion, the crustal structure of the reference model has been corrected by using the 3SMAC model (Nataf & Ricard, 1996).

6.4.2 Local shear wavespeed models

The local phase dispersion curves are assembled from the multi-mode dispersion maps at 2° grid points along both longitude and latitude, and then are inverted for local shear wavespeed structures.

The number of iterations required for the satisfactory convergence achieved through the inversion varies with the choice of a reference model and *a priori* information. With an appropriate choice of the reference model to start the inversion in (6.27), the model can converge in the first few iterations, since the nonlinearity between the phase dispersion and shear wavespeed is quite moderate. However, typical 1-D shear wavespeed models in the Australian region are generally quite far from the PREM, showing a quite large velocity perturbation in the upper 250 km.

We therefore repeat the whole process of inversion (6.27) using updated reference models which are derived from the previous inversions. This allows us a good recovery of large velocity perturbations which sometimes reach about $\pm 10\%$ from the PREM. In most



Fig. 6.15. The locations of local shear wavespeed models in Fig 6.16 are shown as purple dots and those in Fig 6.17 as red triangles, together with the Rayleigh wave phase speed map of fundamental mode at 100 s.

cases, the shear wavespeed models converges at satisfactory levels within the first 5 to 10 iterations.

We display some examples of the local 1-D shear wavespeed profiles along a latitudinal line $(24^{\circ}S)$ and a longitudinal line $(130^{\circ}E)$ for the locations shown in Fig 6.15.

The 1-D shear wavespeed models for Riz-GCiz along the 24°S latitude across the middle of the Australian continent is shown in Fig 6.16. We can see significant variations in the shear wavespeed structure across the continent, especially above 300 km. In the western region, there are prominent high velocity anomalies in the upper 300 km corresponding to the Proterozoic and Archaean cratonic region in the western Australia, whereas a conspicuous set of low velocity anomalies are seen from 145° to the east representing slow velocities in the Phanerozoic region of the eastern Australia and the Coral Sea.

The other set of 1-D shear wavespeed models along the N-S lines at 130° E longitude are displayed in Fig 6.17. We can again see the high velocity anomalies near the centre of the Australian continent in the upper 300 km. Along this longitude, there is a continent-ocean boundary around 32° S latitude, and we can identify the slower velocities to the south of the boundary and the faster anomalies to the north suggesting the rapid changes in the shear wavespeed at the boundary. The slower anomalies around 15° S correspond to a



Fig. 6.16. Local shear wavespeed models along the 24° south latitude with varying longitude. Locations of these models are indicated as purple dots in Fig 6.15.



Fig. 6.17. Local shear wavespeed models along the 130° east longitude with varying latitude. Locations of these models are indicated as red triangles in Fig 6.15.



Fig. 6.18. Resolution kernels for a shear wave speed structure beneath $(24^{\circ}S, 140^{\circ}E)$.

patch of slower wavespeeds near the northern edge of Australia which can be seen in the phase speed maps (Fig 6.10, 6.15).

The resolution kernels of a typical 1-D shear wavespeed profile is shown in Fig 6.18. It is apparent that the shallower parts of the 1-D structure from 100 to 250 km are comparatively well constrained, whereas the deeper part of the upper mantle are not well resolved even though we include information of up to the third higher mode. This is mainly due to the smaller absolute sensitivity of the higher modes to the shear wavespeed structure compared to that of the fundamental mode. Still we can see some sensitivity around 400 km depth where it is almost impossible to resolve only with the fundamental mode at the maximum period of 150 s used in this study.

6.5 3-D models in the Australian region

Repeating the inversions for the local shear wavespeed profile across the whole region, we can obtain the final 3-D models. Since the minimum scale length of the lateral heterogeneity that can be resolved by our phase speed inversion are longer than a few hundred kilometers, corresponding to the minimum wavelength used in this study, we assemble the local phase dispersion curves for each $2^{\circ} \times 2^{\circ}$ cell and invert them for the local shear wavespeed structure. We have obtained five types of 3-D shear wavespeed models from the corresponding sets of phase speed models in Fig 6.7.

In this section, we mainly focus on the comparison of these 3-D models, especially how the models can be improved by considering the influence zone of surface wave paths. A 3-D model derived from a 2-stage approach is also displayed for the comparison of the models derived from the different inversion techniques. Full sets of these 3-D models are shown in Appendix E.

6.5.1 Comparison of 3-D models I: two-stage and three-stage approach

Although the three-stage approach for surface wave tomography is able to provide us with a number of benefits for improving 3-D models, the process of obtaining a final 3-D model is rather indirect compared to the conventional two-stage approach. Therefore, comparing the 3-D models derived from these two different approach is helpful for assessing how the differences in the inverion processes affect the final 3-D models.

In this section, to compare the models derived from different types of inversion scheme, we first obtain 3-D models using a form of two-stage inversion scheme with the direct use of the path-specific 1-D profiles of Debayle & Kennett (2002) that are used to estimate multi-mode phase speeds in this study. Following arguments in the appendix to Debayle & Kennett (2000a), a linear relation for a path-average shear wavespeed perturbation at a particular depth z can be given as,

$$\left\langle \frac{\delta\beta}{\beta} \right\rangle^{obs} \bigg|_{z} = \frac{1}{\Delta} \int_{g.c.} \frac{\delta\beta(s)}{\beta} \bigg|_{z} ds.$$
(6.31)

This linear relation can be solved in the same way as the phase speed inversion explained in the preceding section, using the LSQR algorithm for a damped least-squares problem (6.22).

The 3-D model derived from the two-stage inversion scheme shows similar behaviour in the trade-off between the misfit and model norm, and thus we choose an appropriate damping parameter which shows tradeoff behaviour similar to Fig 6.8. Variance reductions achieved through the direct use of the path-average 1-D models are more than 70 % for the models above 200 km, whereas, for the models below 250 km, the variance reduction are achieved around 40 %, but the model explain the data quite well with respect to the estimated errors.

The 3-D models derived from the two-stage and three-stage approach are displayed in Fig 6.19. The three-stage model has been derived from a set of phase speed models GC0, whereas the two-stage models are directly retrieved from the path-specific 1-D profiles.


Fig. 6.19. 3-D shear wavespeed models in the Australian region derived from the two-stage approach (left column) and from the three-stage approach (right column) at the depth from 100 to 250 km with 50 km increment. The three-stage model is the GC0 model which is derived from the great-circle approximation. Reference wavespeeds are 4.41 km/s at 100 km, 4.43 km/s at 150 km, 4.51 km/s at 200 km, 4.61 km/s at 250 km.

Despite of the intrinsic differences in the inversion processes, the final 3-D models are extremely well correlated. The geographical correlation coefficients of these velocity structures exceed 0.95 at all depths. Resolution maps for the two-stage models are similar to those shown in Fig 6.14 (a,c,e) depending on the depth. Although the process of recovering these models are different, the underlying assumptions are the same. That is, all the surface wave paths are assumed to lie along the corresponding great-circle and no finite-frequency effects are considered. The similarity of these models indicate that the roundabout route to reach the final 3-D model in the three-stage approach (i.e., the use of the intermediary of the phase speed maps) does not bring in any noticeable error in the final models in the intermediate frequency range used in this study. This fact also ensures the profit from the three-stage approach, which is more convenient to incorporate with various phenomena of surface wave propagation such as the frequency dependent offgreat-circle propagation and the influence zone around surface wave paths to take account of the finite-frequency effects.

It should be note that the appearance of these 3-D models are different from that of Debayle & Kennett (2000a) in Fig 1.2 because of the differences of the data set. However the 3-D models in Fig 6.19 are quite similar to that of Debayle & Kennett (2002) whose date sets is utilised in this chapter, despite the different inversion technique.

6.5.2 Comparison of 3-D models II: three-stage models

Using the five sets of phase speed models in Fig 6.10, we obtain five types of corresponding 3-D shear wavespeed models. The shear wavespeed models at 150 km depth for the different types of 3-D models are shown in Fig 6.20. The major features of these models are quite similar to those of the corresponding phase speed maps in Fig 6.10, sharing the similar patterns of the fast and slow wavespeed anomalies.

The models based simply on the geometrical ray theory shown in Fig 6.20 (a) and (b) have rather similar patterns of heterogeneities and their amplitude. This suggests that the process of model update only considering ray path bending does not have a significant impact on the final models at the frequency range in this study. In other words, the assumption of the surface wave propagation along the great-circle does work quite well at the period longer than 40 seconds. This can also be expected from the results of two-point ray shooting experiment shown in section 2.7, which show that the actual ray paths with the minimum travel times in the sense of Fermat's principle, are not very far away from the corresponding great-circles.

On the other hand, the models derived including the influence zone around the surface wave path (Fig 6.20 (c), (d) and (e)), share some similar features of somewhat smoothed



Fig. 6.20. Five types of shear wave speed models at 150 km derived from the corresponding phase speed maps in Fig 6.10 based on the three-stage approach. Reference wavespeeds are the same as Fig 6.19.

heterogeneities, regardless of the inclusion of effects from the off-great-circle propagation. In this section, we make some more comparisons of these models especially focusing on how the influence zone affects the models. Thus, we do not present a comparison of the models GC0 and Ray-GC0 since both models are simply based just on the geometrical ray theory (Full sets of these models are displayed in Appendix E).

6.5.2.1 GC0 and Riz-GC0

We display two sets of shear wavespeed models in Fig 6.21, $GC\theta$ derived from the greatcircle approximation and Riz- $GC\theta$ updated from $GC\theta$ taking both the off-great-circle propagation and the influence zone into account.

We can see that some small patch-like features in the model $GC\theta$ are smoothed out in the *Riz-GC0*. This is especially apparent near the faster velocity anomalies beneath the Proterozoic blocks in the centre of the Australian Continent, and also in the slower velocity regions in the Coral and Tasman Sea to the east off-shore of Australia.

The smoothing effects in the finite-frequency model Riz-GC0 become clearer in the deeper parts of the mantle (≥ 150 km). This can be attributed to the fact that the structures at these depths are constrained mainly by the higher modes and long-period fundamental modes whose influence zones are wider than those for the short-period fundamental mode.

The effect of the smoothing caused by the finite-width of the rays is also preserved in the cross sections shown in Fig 6.22, especially in the region where there are noticeable smoothing in the heterogeneity in the map projections in Fig 6.21, i.e., beneath central Australia and the oceanic region to the east of Australia.

In either model, we can clearly see higher velocity anomalies down to depths of 200 to 250 km beneath the middle and western parts of Australia, corresponding to the continental lithosphere of the Australian Continent. The depth of the root of such continental lithosphere can be estimated from the largest velocity gradient in the wavespeed profiles. In a region just beneath the Proterozoic blocks in the central Australia (around 20°S and 132°), the continental lithosphere seem to reach 300 km. This is quite consistent with the results of Simons et al. (1999) who have also suggested that the higher wavespeed anomaly in this region is likely to persist to depth over 300 km.

One of the major difference from the models of Simons et al. (1999) is that our models do not show extreme high wavespeed anomalies in the eastern Australia beneath 250 km. At this depth, the wavespeed perturbation from the PREM become very small beneath the Australian Continent and the maximum perturbation do not exceed $\pm 2\%$. The cause of the differences can be attributed to the differences in the data analysis, that is, the



Fig. 6.21. 3-D shear wavespeed models derived from the great-circle approximation based on the geometrical ray theory ($GC\theta$ in the left column) and those updated from $GC\theta$ including the effects of off-great-circle propagation and the influence zone (Riz- $GC\theta$ in the right column) at the depth from 100 to 250 km with 50 km increment. Reference wavespeeds are the same as Fig 6.19.



Fig. 6.22. Cross sections of 3-D shear wavespeed models in Fig 6.21. (a) N-S cross sections through varying longitudes for the model GC0. (b) E-W cross sections through various latitudes for the model GC0. (c) Same as (a) but for the model *Riz-GC0*. (d) Same as (b) but for the model *Riz-GC0*.

waveform fitting procedures are essentially different. Our data set of path-specific 1-D models of Debayle & Kennett (2002) have been derived from fitting cross-correlograms as secondary observables as we have explained in the section 6.2, whereas the Simons et al. (1999) have used a procedure of fitting the multi-mode waveforms directly for path-specific 1-D models (e.g., Nolet, 1990), which is more sensitive to the choice of initial models to start the inversion (Hiyoshi, 2001).

6.5.2.2 GCiz and Riz-GCiz

We next compare a 3-D model GCiz derived from the inversion including the influence zone around the great-circle and an updated model Riz-GCiz from the GCiz considering the effects of both off-great-circle paths and the influence zone (Fig 6.23 and 6.24).

Since both sets of models include the effects of the finite-frequency, both models contain smoothed characters caused by the use of the influence zone. The patterns of heterogeneities in these models are very similar and there is little remarkable differences between these two sets of models. Also, as we have already seen in Fig 6.20, two sets of the finitefrequency models, Riz-GC0 in Fig 6.21 and Riz-GCiz in Fig 6.23, are extremely similar at all depths, even though these models are updated from different reference heterogeneous models. These facts suggest that the process of global iteration (i.e., the update process for the models) using the influence zone gives quite consistent results even when we use different initial models.

Higher wavespeed anomalies in the western Australia just beneath the NWAO station, corresponding to the root of the Archaean craton seem to get fainter at a depth of 250 km of *Riz-GCiz* (Fig 6.23 (h); Fig 6.24 (d) at $30^{\circ}S$). Similar features of higher wavespeed anomaly in this region at this depth can be seen in the comparisons of models *GC0* and *Riz-GC0* (Fig 6.21 (d) and (h)).

Earlier studies (Simons et al., 1999; Debayle and Kennett, 2000a) have shown high wavespeed anomalies at this depth, similar to models GC0 and GCiz. We may say the weakened higher wavespeed anomaly in this region in the models Riz-GC0 and Riz-GCiz can be caused by the inclusion of the effects of ray-path bending and the influence zone about the paths.

However, with the presently available path coverage, this region has limited resolution because of the sparse path coverage. Therefore we need to be careful about discussing the structure in the western blocks of the Australian Continent. Even so, the ray tracing experiments shown in chapter 2 suggest that conspicuous ray path deviations from the great-circle are likely to appear in the paths from the Tonga-Kermadec region to the NWAO station. Therefore, the use of the ray tracing in phase speed structures should play a role to suppress some undesirable effects caused by large ray path deviations at the NWAO station.

6.6 Discussion

In this chapter we have explicitly formulated the three-stage inversion which enables us to incorporate off-great-circle propagation and the influence zone for surface wave paths



Fig. 6.23. 3-D shear wavespeed models derived from the great-circle approximation with the influence zone (GCiz in the left column) and those updated from GCiz including the effects of off-great-circle propagation and the influence zone (Riz-GCiz in the right column) at the depth from 100 to 250 km with 50 km increment. Reference wavespeeds are the same as Fig 6.19.



Fig. 6.24. Cross sections of 3-D shear wavespeed models in Fig 6.23. (a) N-S cross sections through varying longitudes for the model GCiz. (b) E-W cross sections through various latitudes for the model GCiz. (c) Same as (a) but for the model Riz-GCiz. (d) Same as (b) but for the model Riz-GCiz.

in the frequency and mode domains. We have also applied the new technique to the Australian region and have displayed several styles of new 3-D Australian models.

The three-stage inversion scheme is a rather indirect approach for recovering the 3-D shear wavespeed models, compared to the conventional two-stage approach used in most regional studies (e.g., Nolet, 1990, Debayle & Kennett, 2000a). However, a model derived from the great-circle approximation ($GC\theta$) using the three-stage approach shows

6.6 Discussion

quite similar features to a 3-D model obtained from the 2-stage inversion scheme with the direct use of path-specific 1-D models. This suggests that the use of phase speed maps as intermediaries does not cause significant errors in the final 3-D model.

The advantage of the three-stage approach is that various types of information can be brought together in a common formulation. The polarization anomalies due to ray path bending and the influence zone that takes account of finite frequency effects of surface waves can be efficiently brought together by working with multi-mode phase speed maps at each frequency.

In practical applications, the three-stage inversion scheme requires the computation of a number of phase speed maps to better constrain the final 3-D models, followed by inversions for local shear wavespeed models. Therefore, in total, the three-stage approach requires more computation than for the two-stage process, even though our approach is still efficient enough to treat complicated phenomena of off-great-circle propagation and the finite frequency effects.

The most time consuming process is the computation of the inversion kernels (6.16) in the second stage. In particular, when we incorporate ray tracing for all the paths in phase speed maps, the computation time becomes almost doubled compared to that with the great-circle approximation. The inclusion of the influence zone further requires us to calculate spatial integrations (rather than line integrations) over a space that is covered by the influence zone. The total amount of computation for the spatial integration depends largely on the mode and frequency, because the width of the influence zone to be integrated varies significantly with these factors. In the practical modeling in this thesis, the computation for the models GCiz requires more than 5 times than that for GC0, and for Riz-GC0 and Riz-GCiz, they require more than 10 times of computation time than GC0.

Still, the concept of the influence zone allows us an approximate treatment of the finitefrequency effects rather than needing to work with a more rigorous formulation including the effects of scattering from much wider region far away from the central ray, which will require much more computations than our approach (see chapter 7).

There are still some points that can be improved in future studies, e.g., spatial parameterisation to treat the varying sizes of the geographic cells, appropriate crustal corrections to better compute phase speeds at shorter period ranges (around 40 s). We will discuss future improvements for our technique of surface wave tomography in chapter 8.

Beyond ray theoretical tomography

7.1 Introduction

In the preceding chapters, we have investigated a form of 2-D sensitivity kernels for surface wave phase based on the simple concept of the influence zone defined in chapter 4. These finite-width kernels have been applied to the practical tomographic inversion for surface waves in chapter 6. However, as we have discussed in chapter 4, the influence zone does not encompass the entire region which could give rise to scattering and diffraction of surface waves. In this chapter, we further investigate the 2-D sensitivity kernels for surface wave phase and amplitude based on surface wave potential theory. This leads to a treatment of scalar-wave type propagation of the surface waves in laterally heterogeneous structure based on Born and Rytov approximations.

Studies on the diffraction and scattering of seismic waves have been tackled by many researchers based on single scattering theories using Born and Rytov approximations. Tomographic inversion considering the diffraction effects of seismic wave propagation was introduced by Devaney (1984) in the context of exploration geophysics based on his method of diffraction tomography for ultrasound waves. Wielandt (1987) discussed the effects of diffraction on body wave travel times, and showed that diffracted waves could have noticeable amplitudes and that the simple ray approximation breaks down in such circumstances.

For surface waves, Yomogida & Aki (1987) initiated tomographic inversion with finitewidth kernels derived from their asymptotic formulation in terms of Gaussian beams for surface waves (Yomogida, 1985; Yomogida & Aki, 1985). They produced 2-D sensitivity kernels for phase speed structure based on the Born and Rytov approximations and applied them to the reconstruction of phase speed maps in the Pacific region using both the phase and amplitude of surface waves. Their approach is based on the assumption of single-mode 146

7.1 Introduction

surface wave propagation using a 2-D scalar wave equation. Tanimoto (1990b) tackled this kind of problems for surface waves with an alternative approach using potential theory taking account of the inter-mode conversion of surface waves in a phase speed domain. This potential representation was further extended by Tromp & Dahlen (1993) to be able to accommodate local radial eigenfunctions.

Červený & Soares (1992) proposed an efficient way to estimate the Fresnel volume around body-wave paths based on the paraxial ray theory. Subsequently Vasco et al. (1995) applied such paraxial Fresnel volumes to reconstruct seismic images from borehole data, and they showed that tomographic inversion with their time-consuming exact kernels and with more efficient paraxial kernels give comparable results.

Sensitivity kernels for body wave travel times at finite frequency have been proposed by many researchers (Luo & Shuster, 1991; Woodward, 1992; Yomogida, 1992; Vasco & Majer, 1993; Li & Tanimoto, 1993; Marquering, Dahlen & Nolet, 1999; Dahlen, Hung & Nolet 2000; Zhao, Jordan & Chapman 2000). Li & Tanimoto (1993) have obtained 2-D sensitivity kernels for long-period body waves using asymptotic normal mode coupling, and Li & Romanowicz (1995) applied this method for waveform inversion for a global Earth model. Marquering & Snieder (1995) have proposed sensitivity kernels using surface-wave mode rather than normal mode to calculate mode-branch coupling, based on the mode coupling theory of Kennett (1984) and the first Born scattering theory of Snieder (1986). Snieder & Lomax (1996) have investigated the effects of velocity perturbations in the structure on the phase and amplitude of the scalar wavefield using the Rytov approximation.

Meier et al. (1997) proposed a way to perform inversion for surface waves considering diffraction effects based on the WKBJ approximation and the first Born scattering theory of Snieder & Nolet (1987). Marquering et al. (1998) have also used the linearised scattering theory to calculate 3-D waveform sensitivity kernels. Marquering et al. (1999) calculated body-wave travel time kernels using waveform kernels based upon the surface-wave mode coupling. Dahlen et al. (2000) reformulate the body wave travel time kernels using the Born scattering of body waves; they also adopt the paraxial ray theory to simplify the calculation of time consuming two-point ray shooting. Hung et al. (2000) showed that the kernels with paraxial approximation can be a good representation of ray theoretically "exact" kernels in vertically heterogeneous and laterally homogeneous media. Zhao et al. (2000) proposed body wave travel time kernels based on normal mode theory and suggest that the sensitivity on the central ray path is smaller than the surrounding area but not exactly zero as in the sensitivity kernels derived from the ray theory.

In chapter 4, we have applied the technique of Červený & Soares (1992) for surface waves and obtained paraxial Fresnel areas around the central ray path, and proposed a concept of the *influence zone* in which wavefields are coherent. The major target of the influence zone is slightly different from the other studies on the surface wave scattering because the influence zone of our definition only represents the sampling regions around the central ray path, and does not explain the full effects of scattering and diffraction in the entire region around the path.

In order to investigate how the velocity structure affects the surface wavefield, in this chapter, we first derive general expressions for the sensitivity kernels of surface waves based on the Born and Rytov approximations (e.g., Born & Wolf, 1999) using a surface wave potential representation following Tanimoto (1990b) and Tromp & Dahlen (1993). The general forms of these kernels are similar to those given by Yomogida & Aki (1987), Woodward (1992) and Snieder & Lomax (1996). We then employ asymptotic Green's functions for explicit formulations of the sensitivity kernels. The Born kernels are directly related to the wavefield, which yields a separation of the log amplitude term and the phase of the wavefield. Thus, there are advantages in working with the Rytov kernels for phase speed inversion. We will show some examples of 2-D sensitivity kernels for surface-wave phase speed which allow phase speed inversion for surface waves considering the effects of surface wave scattering and diffraction in a first-order approximation.

7.2 General expressions for sensitivity kernels based on single scattering theory

7.2.1 Born approach

Following Tanimoto (1990b) and Tromp & Dahlen (1993), Love and Rayleigh wave displacement fields can be represented in terms of surface wave potentials U which satisfy a spherical Helmholtz equation. The monochromatic surface wave potential U in laterally heterogeneous media can be expressed as,

$$U(\mathbf{r},\omega|\mathbf{r}_s) = U_0(\mathbf{r},\omega|\mathbf{r}_s) + \delta U(\mathbf{r},\omega|\mathbf{r}_s),\tag{7.1}$$

where $U_0(\mathbf{r}, \omega)$ is a surface wave potential in a background (reference) model at a position \mathbf{r} and frequency ω , and δU is a perturbed potential of surface waves generated by lateral heterogeneity.

The surface wave potential U corresponds to a scalar-type wave and is written as $U = A \exp(i\psi)$, where the amplitude A and the phase ψ are slowly varying functions of locations **r**. Actual surface wavefields in 3-D media should be in a vector form with an appropriate radial eigenfunctions, however, since our major objective in this study is to investigate the effects of the velocity structure around a path on the phase perturbation of surface waves,

such a potential representation allows us a reasonable and efficient treatment of the path effects on phases.

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The surface wave potential U satisfies a spherical Helmholtz equation (Tanimoto, 1990b; Tromp & Dahlen, 1993),

$$\left[\nabla_1^2 + k^2(\mathbf{r},\omega)\right] U(\mathbf{r},\omega|\mathbf{r}_s) = f(\mathbf{r}_s,\omega),\tag{7.2}$$

where ∇_1 is the surface Laplacian, k is the wavenumber which is related to surface-wave phase speed $c(\mathbf{r}, \omega)$ with $k = \omega/c$, and f is a source term at a position \mathbf{r}_s .

Using the reference phase speed $c_0(\mathbf{r}, \omega)$, (7.2) can be modified to,

$$\left[\nabla_1^2 + \frac{\omega^2}{c_0^2(\mathbf{r},\omega)} \frac{c_0^2(\mathbf{r},\omega)}{c^2(\mathbf{r},\omega)}\right] U(\mathbf{r},\omega|\mathbf{r}_s) = f(\mathbf{r}_s,\omega),$$
(7.3)

and thus,

$$\left[\nabla_1^2 + k_0^2(\mathbf{r},\omega)\right] U(\mathbf{r},\omega|\mathbf{r}_s) = k_0^2(\mathbf{r},\omega) \left[1 - \frac{c_0^2(\omega)}{c^2(\mathbf{r},\omega)}\right] U(\mathbf{r},\omega|\mathbf{r}_s) + f(\mathbf{r}_s,\omega),$$
(7.4)

where $k_0(\mathbf{r}, \omega) = \omega/c_0(\mathbf{r}, \omega)$. Note that we allow spatial variation of the reference wavenumber k_0 and the reference phase speed c_0 . The term in brackets of right-hand side in (7.4) can be approximated as,

$$1 - \frac{c_0^2}{c^2} = \frac{c^2 - c_0^2}{c^2} = \frac{2c\delta c - (\delta c)^2}{c^2} \approx \frac{2\delta c}{c}, \quad \text{with} \quad \delta c = c - c_0.$$
(7.5)

Thus (7.4) can be reduced to,

$$\left[\nabla_1^2 + k_0^2\right] U(\mathbf{r}, \omega | \mathbf{r}_s) = \frac{2k_0^2 \delta c}{c} U(\mathbf{r}, \omega | \mathbf{r}_s) + f(\mathbf{r}_s, \omega),$$
(7.6)

where we omit the dependency of phase speed and wavenumber on location \mathbf{r} and on frequency ω . Hereafter we will omit these variables unless otherwise specified.

The reference surface-wave potential U_0 may also be represented by a scalar Helmholtz equation in the reference medium,

$$\left[\nabla_1^2 + k_0^2\right] U_0(\mathbf{r}, \omega | \mathbf{r}_s) = f(\mathbf{r}_s, \omega).$$
(7.7)

We now introduce a scalar Green's function $G(\mathbf{r}, \omega | \mathbf{r}')$ which satisfies,

$$\left[\nabla_1^2 + k_0^2\right] G(\mathbf{r}, \omega | \mathbf{r}') = -\delta(\mathbf{r} - \mathbf{r}'), \tag{7.8}$$

with a boundary condition of outgoing wave radiation as $\hat{\mathbf{r}}$ moves away from $\hat{\mathbf{r}}'$ (Dahlen, 1980). Substituting (7.1) into (7.6) and using (7.7), we obtain

$$\left[\nabla_1^2 + k_0^2\right] \delta U(\mathbf{r}, \omega | \mathbf{r}_s) = \frac{2k_0^2 \delta c}{c} U(\mathbf{r}, \omega | \mathbf{r}_s).$$
(7.9)

With the Green's function defined in (7.8), the perturbed surface-wave potential δU may be expressed as,

$$\delta U(\mathbf{r},\omega|\mathbf{r}_s) = \int \frac{-2k_0^2 \delta c}{c} G(\mathbf{r},\omega|\mathbf{r}') U(\mathbf{r}',\omega|\mathbf{r}_s) d\mathbf{r}'.$$
(7.10)

Replacing U in the right-hand side of (7.10) with a reference surface-wave potential U_0 (the first Born approximation),

$$\delta U(\mathbf{r},\omega|\mathbf{r}_s) = \int \frac{-2k_0^2 \delta c}{c} G(\mathbf{r},\omega|\mathbf{r}') U_0(\mathbf{r}',\omega|\mathbf{r}_s) d\mathbf{r}'$$
(7.11)

$$= \int K_U(\mathbf{r}, \mathbf{r}', \mathbf{r}_s, \omega) \left(\frac{\delta c}{c}\right) d\mathbf{r}'.$$
(7.12)

This is a general form of a linearised equation for surface wave perturbations represented as an spatial integral of the phase speed perturbation.

If we invert surface waveforms for a model parameter $\delta m = \delta c/c$ using (7.12), the general form of the sensitivity kernels K_U for the surface wave potential based on the Born approximation can be represented as,

$$K_U(\mathbf{r}, \mathbf{r}', \mathbf{r}_s, \omega) = \left[\frac{\partial U}{\partial m}\right]_{\omega} = -2k_0^2(\mathbf{r}', \omega)G(\mathbf{r}, \omega|\mathbf{r}')U_0(\mathbf{r}', \omega|\mathbf{r}_s).$$
(7.13)

The integral equation (7.12) manifests that the Born kernel K_U relates the phase speed perturbation to the perturbation of the spectrum of the waveform which may be caused by lateral heterogeneity. In most linearised surface wave tomography for phase speed structure, we generally work with phase and amplitude information separately. In such a case, Rytov's method would provide a more direct relation to the phase and amplitude of surface waves to the model parameters as explained in the next section.

7.2.2 Rytov approach

Now we employ Rytov approximation for obtaining sensitivity kernels for phase and amplitude perturbation. In the Rytov methods, the logarithm of the wavefield ($\Phi = \ln U$) is considered instead of the wavefield itself. We first express the surface wave potential Uas,

$$U(\mathbf{r},\omega|\mathbf{r}_s) = \exp[\Phi(\mathbf{r},\omega|\mathbf{r}_s)]$$
(7.14)

$$= A(\mathbf{r}, \omega | \mathbf{r}_s) \exp(i\psi(\mathbf{r}, \omega | \mathbf{r}_s)), \tag{7.15}$$

where A is the amplitude and ψ is the phase term for the surface wave potential. By taking the logarithm, Φ can be divided into real and imaginary parts,

$$\Phi = \ln A + i\psi,\tag{7.16}$$

where we omit the spatial and frequency dependence. Substituting (7.14) into (7.6) yields,

$$\left[\nabla_1^2 + k_0^2\right] \exp[\Phi] = \frac{2k_0^2 \delta c}{c} \exp[\Phi] + f.$$
(7.17)

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Using the relationship $\nabla_1^2 \exp[\Phi] = \nabla_1^2 \Phi + (\nabla_1 \Phi)^2$, (7.17) can be written as,

$$\nabla_1^2 \Phi + (\nabla_1 \Phi)^2 = -k_0^2 \left(1 - \frac{2\delta c}{c} \right) + f \exp[-\Phi].$$
(7.18)

Now, let us assume a perturbation of Φ ,

$$\Phi = \Phi_0 + \delta \Phi \tag{7.19}$$

where Φ_0 is the logarithm of a reference waveform which will satisfy the equation,

$$\nabla_1^2 \Phi_0 + (\nabla_1 \Phi_0)^2 = -k_0^2 + f \exp[-\Phi].$$
(7.20)

Substituting (7.19) and (7.20) into (7.18) and assuming $\Phi_0 \gg \delta \Phi$ yields,

$$\nabla_1^2 \delta \Phi + 2\nabla_1 \Phi_0 \nabla_1 \delta \Phi = \frac{2k_0^2 \delta c}{c},\tag{7.21}$$

where we neglect a second order term. By assuming $\delta \Phi = P(\mathbf{r}, \omega) \exp[-\Phi_0(\mathbf{r}, \omega)]$, (7.21) can be reduced to a Helmholtz equation as follows,

$$\left[\nabla_1^2 + k^2\right] P(\mathbf{r}, \omega) = \frac{2k_0^2 \delta c}{c} \exp[\Phi_0].$$
(7.22)

With the Green's function introduced in (7.8), P may be expressed as,

$$P(\mathbf{r},\omega|\mathbf{r}_s) = \delta \Phi e^{\Phi_0} = \int \frac{-2k_0^2 \delta c}{c} G(\mathbf{r},\omega|\mathbf{r}') \exp[\Phi_0(\mathbf{r}',\omega|\mathbf{r}_s)].$$
(7.23)

Finally, we obtain an integral equation for $\delta \Phi$,

$$\delta\Phi(\mathbf{r},\omega|\mathbf{r}_s) = \int \frac{-2k_0^2 \delta c}{c} \frac{G(\mathbf{r},\omega|\mathbf{r}')U_0(\mathbf{r}',\omega|\mathbf{r}_s)}{U_0(\mathbf{r},\omega|\mathbf{r}_s)} d\mathbf{r}'$$
(7.24)

$$= \int K_{\Phi}(\mathbf{r}, \mathbf{r}', \mathbf{r}_s, \omega) \left(\frac{\delta c}{c}\right) d\mathbf{r}'.$$
(7.25)

Thus the sensitivity kernels for Φ can be given by,

$$K_{\Phi}(\mathbf{r}, \mathbf{r}', \mathbf{r}_s, \omega) = \left[\frac{\partial \Phi}{\partial m}\right]_{\omega} = -2k_0^2 \frac{G(\mathbf{r}, \omega | \mathbf{r}') U_0(\mathbf{r}', \omega | \mathbf{r}_s)}{U_0(\mathbf{r}, \omega | \mathbf{r}_s)}.$$
(7.26)

From the relationship (7.16), the imaginary part of K_{Φ} relates the phase speed perturbation δc to the phase perturbation $\delta \psi$ caused by lateral heterogeneity whereas the real part corresponds to the sensitivity for the logarithm of the amplitude term A. These are explicitly derived from (7.25),

$$\delta\psi = \int \operatorname{Im}\left\{K_{\Phi}\right\}\left(\frac{\delta c}{c}\right) d\mathbf{r}',\tag{7.27}$$

$$\delta \ln A = \int \operatorname{Re} \left\{ K_{\Phi} \right\} \left(\frac{\delta c}{c} \right) d\mathbf{r}'.$$
(7.28)

In most surface wave tomography, the phase perturbation is first measured from observed seismograms, and perturbations are inverted for phase speed structure. Therefore, the integral equations (7.27) and (7.28) can be used directly for 2-D phase speed inversion based on the phase and amplitude measurements for surface waves. In Section 4, we will show examples of the sensitivity kernels mainly focusing on the imaginary part of the Rytov kernels.

7.3 Representation of sensitivity kernels with asymptotic ray theory

7.3.1 WKBJ approximation

The sensitivity kernels derived in the previous section can be explicitly represented by using WKBJ approximation (e.g., Tromp & Dahlen, 1992; Dahlen & Tromp, 1998). Tromp & Dahlen (1993) obtained the scalar Green's function for (7.8) in a laterally heterogeneous media employing the asymptotic results of Dahlen (1980),

$$G(\mathbf{r},\omega|\mathbf{r}') = \left[\frac{1}{8\pi k(\mathbf{r})J(\mathbf{r},\mathbf{r}')}\right]^{\frac{1}{2}} \exp i\left\{\int_{l} kd\mathbf{r} + \frac{\pi}{4}\right\},\tag{7.29}$$

where J is a geometrical spreading factor ($J = \sin \Delta$ for a homogeneous media, where Δ is a epicentral distance), l represents a ray path from $\mathbf{r'}$ to \mathbf{r} . Note that we omit a term associated with the Maslov index (Tromp & Dahlen, 1992) which represent the number of caustics along a ray path. Here we only consider surface waves passing along minor arcs (i.e., R1 & G1).

Following the WKBJ theory in Tromp & Dahlen (1992), the reference surface wave potential U_0 in (7.13) and (7.26) may be represented as,

$$U_0(\mathbf{r},\omega|\mathbf{r}_s) = \left[\frac{1}{8\pi k(\mathbf{r})J(\mathbf{r},\mathbf{r}_s)}\right]^{\frac{1}{2}} S(\mathbf{r},\mathbf{r}_s,\omega) \exp i\left\{\int_l k d\mathbf{r} + \frac{\pi}{4}\right\},\tag{7.30}$$

where $S(\mathbf{r}, \mathbf{r}_s, \omega)$ is the source term for a moment tensor source radiated toward \mathbf{r} from the source location \mathbf{r}_s . We have employed the same normalisation convention as used in Tromp & Dahlen (1992). Although we have assumed 2-D scalar wave propagation, the source term S still requires eigenfunctions at the source location. We may employ a spherical reference model with appropriate crustal corrections for the source location to calculate the eigenfunctions at the source.

Using (7.29) and (7.30), the explicit form of the sensitivity kernels (7.13) and (7.26) can be derived. For the Born sensitivity kernels,

$$K_U(\mathbf{r}, \mathbf{r}', \mathbf{r}_s, \omega) = \frac{-\omega}{4\pi c_0} \left[\frac{1}{J(\mathbf{r}, \mathbf{r}')J(\mathbf{r}', \mathbf{r}_s)} \right]^{\frac{1}{2}} S(\mathbf{r}, \mathbf{r}_s, \omega) \exp i \left(\int_{l_1} k d\mathbf{r} + \int_{l_2} k d\mathbf{r} + \frac{\pi}{2} \right), \quad (7.31)$$

where l_1 and l_2 show ray paths from \mathbf{r}' to \mathbf{r} and from \mathbf{r}_s to \mathbf{r}' , respectively.

Similarly, the explicit expression for the Rytov kernels can be given by,

$$K_{\Phi}(\mathbf{r}, \mathbf{r}', \mathbf{r}_{s}, \omega) = -\frac{\omega^{2}}{c_{0}^{2}} \left[\frac{1}{2\pi k_{0}} \right]^{\frac{1}{2}} \left[\frac{J(\mathbf{r}, \mathbf{r}_{s})}{J(\mathbf{r}, \mathbf{r}')J(\mathbf{r}', \mathbf{r}_{s})} \right]^{\frac{1}{2}} \frac{S(\mathbf{r}', \mathbf{r}_{s}, \omega)}{S(\mathbf{r}, \mathbf{r}_{s}, \omega)} \times \exp i \left(\int_{l_{1}} k d\mathbf{r} + \int_{l_{2}} k d\mathbf{r} - \int_{l_{0}} k d\mathbf{r} + \frac{\pi}{4} \right),$$
(7.32)

where l_0 represents a ray path from \mathbf{r}_s to \mathbf{r} . For practical calculations for these kernels, we may evaluate the geometrical spreading J by $J = \sin X$, where X is the distance along a ray path. The integrations of wavenumbers along ray paths require a number of twopoint ray calculations in laterally heterogeneous structure. Although this would provide ray theoretically "exact" sensitivity kernels, it demands a very heavy computation. This computation can be significantly reduced if we employ the paraxial ray approximation as described in the next section.

7.3.2 Paraxial ray approximation

The exponential term in (7.32) can be replaced by,

$$\exp i\left(\psi_1 + \psi_2 - \psi_0 + \frac{\pi}{4}\right),\tag{7.33}$$

where $\psi = \int k d\mathbf{r}$. This phase term can be efficiently derived from the paraxial Fresnel-area ray tracing in the ray centered coordinate system (s, n) developed in chapter 4

$$\delta\psi_F = \psi_1 + \psi_2 - \psi_0 = \frac{1}{2}n^2 M(s), \tag{7.34}$$

where M(s) is taken from equation (4.29) in chapter 4. Using (7.34), we can estimate the paraxial Rytov kernels with just one two-point ray calculation for a central ray and subsequent calculation of dynamic ray tracing along the ray from the source to receiver and from the receiver to source.

The geometrical spreading factors in (7.32) can be estimated in a similar way to that explained in the previous section, but we may need to replace the epicentral distance along a ray segment with its great-circle distance, since we do not know the actual path length for each ray segment without two-point shooting. This approximation works quite well if we only work with a smaller region around the central rays or if the lateral heterogeneity is not too strong and hence ray path deviation is not significant.

The phase term in (7.33) provides an interesting feature of the ray theoretical sensitivity kernels. Just on the central ray, a distance from the central ray n in (7.34) is zero, hence $\delta\psi_F = 0$. Thus, the exponential term in (7.32) is always $\exp i(\pi/4)$ on the central ray path and the sensitivity for the surface wave phase will not be zero on the ray. On the other hand, the ray theoretical 3-D sensitivity kernels for body-wave travel time (e.g., Dahlen et al., 2000) shrinks to zero on the central ray because it does not include this term, and thus the imaginary part of the exponential term becomes zero, resulting in the zero sensitivity for body-wave travel times on the ray.

7.4 2-D sensitivity kernels for surface-wave phase speed structure

In this section, we display examples of sensitivity kernels derived from the Rytov approximation which correspond to the Fréchet derivatives for phase and amplitude perturbation of surface waves and can be directly applied to inversions for phase speed maps.

7.4.1 Sensitivity kernels in a homogeneous model

We first display sensitivity kernels in laterally homogeneous reference model for a path to CAN station from a source near Vanuatu. Fig 7.1 displays the imaginary and real parts of the Rytov sensitivity kernels, which corresponds to Fréchet derivatives for the phase and amplitude variation, respectively, at periods of 40 and 70 seconds without the effects of radiation from the source. These kernels are ray theoretically "exact" kernels based on the WKBJ approximation without the use of paraxial approximation. They are calculated by assuming the source term in (7.32) to be such that $S(\mathbf{r}', \mathbf{r}_s, \omega)/S(\mathbf{r}, \mathbf{r}_s, \omega) = 1$, and thus there is no effect of source radiation. The reference phase speeds are 3.95 km/s at 40 seconds and 4.06 km/s at 70 seconds. Up to the fifth Fresnel zone is shown in these maps. In the phase kernels (left-side figures in Fig 7.1), the bluish area corresponds to the odd-order Fresnel zones (first, third and fifth) and reddish to the even-order Fresnel zones (second and fourth).

It should be noted that some degree of sensitivity exists even outside the fifth Fresnel zone for a monochromatic surface wave. Such side-lobes of the sensitivity kernels can be canceled out when we consider the band-limited kernels and only the sensitivities around the lower-order Fresnel zones remain (Woodward, 1992), although investigating such kernels is not the objective of this study. It is worth noting that the longer paths corresponding to the outer oscillations of the sensitivity kernels correspond to time shifts of more than half the period. For a monochromatic wave the 2π ambiguity in phase maps them into the same oscillation. But, in a real seismogram composed of a superposition of many frequencies, the influence of the outer lobes will shift to later cycles in the waveform.

In both phase and amplitude kernels, we see that maximum sensitivities do not exist on the centre path, but not 0 there, unlike 3-D sensitivity kernels (banana-doughnuts kernels) for body waves based on the ray theory (e.g., Yomogida, 1992; Dahlen et al., 2000; Hung et al., 2000). Such a feature of the kernels in Fig 7.1 is rather closer to the 2-D sensitivity kernels for body waves (e.g., Li & Tanimoto, 1993; Li & Romanowicz, 1995).





Fig. 7.1. Imaginary (left) and real part of the Rytov sensitivity kernels in a homogeneous reference model for a path from Vanuatu to CAN station at 40 (top) and 70 seconds (bottom). The radiation effects at the source is not considered in these kernels, and areas up to the fifth Fresnel zone are displayed. The imaginary part of the Rytov kernels corresponds to the sensitivity kernels for phase variation of Rayleigh waves, whilst the real part corresponds to amplitude variation. Red dotted line is the great-circle. The reference phase speeds are 3.95 km/s at 40 seconds and 4.06 km/s at 70 seconds.



Fig. 7.2. Cross profiles of sensitivity kernels in Fig 7.1 at the centre of the path. Phase kernel is shown in a dotted line and amplitude kernels in a solid line.

The sensitivity varies along the path and the largest sensitivities are seen near the source and receiver locations. The longer the periods, the wider is the width of the Fresnel zones and the maximum sensitivity is somewhat smaller than the shorter period.

The cross profiles of the Rytov kernels in Fig 7.1 at the middle of the path as a function of the distance from the central path are displayed in Fig 7.2. Since no effects of lateral heterogeneity and source radiation are considered, the sensitivities are symmetric with respect to the central path.

Next we investigate the effects of the source radiation on the sensitivity kernels. The Rayleigh-wave radiation pattern calculated for a source in Vanuatu at depth of 50 km is shown in Fig 7.3. The rest of the Rytov kernels with radiation effects shown in this section are calculated with this radiation pattern.

In order to check source radiation effects, we display the Rytov kernels for two paths with different azimuth at the source in Fig 7.4. The sensitivity patterns for a path to CAN



Fig. 7.3. Rayleigh-wave radiation pattern for the source parameters used for the calculations of sensitivity kernels. The directions for two stations, CAN and NWAO, are indicated with black dots.

exhibit an asymmetric strength of the sensitivity due to the effects of source radiation. This is clearly seen in the cross profiles of the sensitivity across the path at the middle of the path in Fig 7.5 (top). On the other hand, for a path to NWAO station, which is located in the direction of the maximum source radiation, the cross profiles at the middle of the path (Fig 7.5 bottom) do not show a conspicuous effect of radiation, and are still symmetric with respect to the central path.

In either case, the oscillation cycle of the kernels as a function of the distance from the central path (Fig 7.5) is not affected by the source radiation, suggesting that the elliptical patterns of the Fresnel zones simply depend on the background phase speed structure and that the source radiation only affects the magnitude and the polarity of the sensitivity.

The opposite polarity of the sensitivity kernels due to the initial phase differences of the source radiation is seen in the north-eastern side of the source. Fig 7.4 also suggests that the sensitivity becomes broader for the longer path (i.e., a path for NWAO). Furthermore, we can see that the longer path tend to have somewhat smaller sensitivity in the middle of the path, compared to the shorter path to CAN station.

7.4.2 Sensitivity kernels with paraxial approximation

So far we have displayed sensitivity kernels which are ray theoretically "exact" based on the WKBJ theory. Now, we examine the kernels with the paraxial ray approximation, which is useful for reducing the computation time especially for these kernels in heterogeneous structure.

Examples of the ray theoretically "exact" and paraxial kernels for fundamental mode



Fig. 7.4. Rayleigh wave sensitivity kernels at 70 seconds in a homogeneous model with the effects of source radiation for paths to CAN and to NWAO stations. The reference phase speed is 4.06 km/s. Up to the fifth Fresnel zone is shown. Red dotted line is the great-circle.

Rayleigh waves at 70 seconds in a homogeneous media are shown in Fig 7.6. We only display the imaginary part of the Rytov kernels that corresponds to the kernels for surface wave phase with up to the second Fresnel zone.

It is apparent that the paraxial kernels are a good approximation of the exact kernels, except near the source and receiver. The kernels using the paraxial approximation cannot be derived across the source and receiver positions, because the term M(s) in (7.34), which is derived from the Fresnel-area ray tracing, can be evaluated only for the locations between the source and receiver.



Fig. 7.5. Cross profiles of sensitivity kernels in Fig 7.4 at the centre of the path as a function of distance from the central ray. Phase kernel is shown in a dotted line and amplitude kernels in a solid line.

Since the paraxial ray approximation is only valid near the central ray path, the differences between the exact and paraxial kernels become more noticeable in the outer region. Nevertheless, the overall features of the exact kernels within the first Fresnel zones are represented very well by the paraxial kernels. This is especially true near the middle of the path and the cross profiles of these exact and paraxial kernels in Fig 7.7 (top) are almost identical for both paths to CAN and NWAO.

7.4.3 Sensitivity kernels in phase speed models

One of the significant advantage of the paraxial ray approximation is that we can easily estimate the Fresnel areas in laterally heterogeneous media. We investigate the "exact" and paraxial sensitivity kernels in a fundamental mode Rayleigh-wave phase speed model



Fig. 7.6. Ray theoretically "exact" (left) and paraxial (right) sensitivity kernels for fundamental mode Rayleigh waves at 70 seconds in a homogeneous model for paths to CAN (top) and NWAO (bottom) stations. Only the imaginary part of the Rytov kernel with up to the second Fresnel zone is shown. Red dotted line is the great-circle. All kernels include the effects of source radiation.

at 70 s derived in chapter 6 (Fig 7.8). The exact kernels are calculated by employing very time consuming two-point shooting for each point with a grid interval of 0.25° .

For a path to CAN station in Fig 7.9 (top), the exact and paraxial kernels are similar and they also resemble the kernels in a homogeneous medium in Fig 7.6 (top). The ray path in this heterogeneous model is not appreciably different from the corresponding great-circle.



Fig. 7.7. Cross profiles of the imaginary part of the Rytov sensitivity kernels across a path at the middle of the central ray (top) for homogeneous media in Fig 7.6 and (bottom) for heterogeneous media in Fig 7.9. The red solid line is the ray theoretically "exact" kernel and the black dotted line is the paraxial kernel.

The similarity of these kernels is also seen in Fig 7.7 (left), suggesting that there is little effect of the lateral heterogeneity on the sensitivity kernels for this path. This is mainly because that the velocity gradients across the path are smooth around this ray path.

On the contrary, for a path to NWAO in Fig 7.9 (bottom), there are differences in the exact and paraxial kernels. This is also clearly seen in the cross profiles of these kernels in Fig 7.7 (bottom-right), and we can see the narrower width for the paraxial kernel than the exact kernel.

Since the paraxial ray theory is relying solely on the velocity gradient along the central ray path to represent the behaviour of the neighbouring rays, the width of the paraxial Fresnel areas is simply determined by the velocity gradients on the ray path. Therefore, the narrower width of the paraxial kernel suggest that the velocity gradient along the path for NWAO may be too large to be treated with the paraxial ray theory. It is also apparent



Fig. 7.8. Phase speed map *Riz-GCiz* in chapter 6 for a fundamental mode Rayleigh wave at 70 seconds.

that the shape of the Fresnel zones for the paraxial kernels for NWAO station are slightly distorted at some part, implying the existence of a locally strong velocity gradient on the central path where the width of the Fresnel zone suddenly shrinks (e.g., near the crossing point of a ray to NWAO and the eastern margin of the Australian continent in Fig 7.9 (bottom-right)).

We can see that the width of the exact kernel in Fig 7.7 (bottom-right, red curve) for NWAO station is not symmetric, i.e., the left-hand side (corresponding to the northernhalf of the kernels in Fig 7.9 (bottom) for NWAO station) of the cross profile of the sensitivity kernel is slightly elongated compared to the right-hand side with respect to the central ray path. This implies differences in the distribution of velocity gradient to the northern and southern sides with respect to the central path to NWAO, although the paraxial kernels are always symmetric with respect to the central ray paths since, in the paraxial ray theory the width of the kernels depend simply on the velocity gradient on the central ray. Note that the asymmetric amplitudes in the kernels in Fig 7.7 (left) for CAN station are caused by the effects of source radiation, not by the differences in velocity distribution in the structure.



Fig. 7.9. Ray theoretically "exact" (left) and paraxial (right) sensitivity kernels for fundamental mode Rayleigh waves at 70 seconds in a Rayleigh-wave phase speed model in Fig 7.8 for paths to CAN (top) and NWAO (bottom) stations. Only the imaginary part of the Rytov kernel with up to the second Fresnel zone is shown, red dotted line is the great-circle and yellow solid line the corresponding ray path. All kernels include the effects of source radiation.

This particular example clearly shows how the paraxial ray approximation begin to break down. In order to obtain a rigorous sensitivity kernels in a heterogeneous medium considering the scattering and diffraction effects from the outside of the influence zone, time consuming exact computations for the "exact" sensitivity kernels will be essential for the practical application for surface wave tomography. However, the exact and paraxial kernels for the path to NWAO are still identical near the central ray path, which includes the region of the influence zone introduced in chapter 4. In other words, the paraxial ray approximation is still useful even for a path that passes through moderately heterogeneous regions as in the path to NWAO in Fig 7.9, as long as we consider a very narrow region around the central ray path, such as the influence zone utilized in the tomographic studies in chapter 6.

7.5 Discussion

We derived 2-D sensitivity kernels for surface wave phase speed structures based on single scattering theory with the Born and Rytov approximation using surface wave potential representation. The sensitivity kernels shown in this chapter will be helpful to take account of the effects of scattered and diffracted waves from a wide region around the path, which cannot be treated with the influence zone.

The Rytov kernels for NWAO station in the laterally heterogeneous structure also imply the limit of the paraxial ray approach for constructing the sensitivity kernels in phase speed maps. Such a paraxial ray approximation has also been used in the recent development of the three-dimensional sensitivity kernels (Dahlen et al., 2000; Hung et al., 2000; Zhao et al., 2000), although these are calculated in vertically heterogeneous but laterally homogeneous models, which correspond to the kernels in a homogeneous model in this study.

Since we have considered only monochromatic waves at particular frequency, sensitivity kernels extend over considerably wider region with very little decrease in the magnitude of the sensitivity away from the central ray path. When we consider the band-limited finite frequency kernels, the side-lobes in each monochromatic kernels are canceled out due to destructive interference, and only the sensitivities around the lower-order Fresnel zones will be remained (e.g., Woodward, 1992).

It should be noted that both the Born and Rytov kernels obtained in this study are based on the surface wave potential which corresponds to a scalar wave representation. Thus, we cannot consider any mode-branch coupling nor the directional dependency of the scattered waves. In order to consider these effects, we may need to work with threedimensional shear wavespeed models as in Marquering et al. (1998, 1999) rather than with phase speed models, although it would then be more complicated to incorporate the effects of off-great-circle propagation (cf. Kennett, 1998a).

The sensitivity kernels obtained in this study can be calculated quite efficiently except for the "exact" kernels in laterally heterogeneous models. For example, it took around 15 minutes to compute the "exact" kernels in Fig 7.9 for NWAO station (distance is around 48 degree) in a phase speed map with 0.25° grids, whereas all the other kernels (the paraxial

$7.5 \ Discussion$

of the geometrical ray theory and the influence zone.

Summary and prospects for future study

8.1 Summary of the thesis

Throughout this thesis, we have investigated various aspects of surface wave propagation and inversion. The main emphasis has been placed on the development of new techniques for surface wave analysis and their application to surface wave tomography, which make it possible to extend the conventional methods of tomographic inversion of surface waves.

A new approach for measuring multi-mode dispersion from a single seismogram was investigated in chapter 3 employing a fully non-linear waveform inversion scheme with the neighbourhood algorithm of Sambridge (1999a). The concept that the path-specific 1-D models can be a good representation of the multi-mode dispersion for the first a few modes became the basis of the use of multi-mode phase speed maps as an intermediary for the reconstruction of 3-D shear wavespeed models.

In chapter 4, we investigated the zone of influence for surface wave path using a hybrid ray tracing technique, *Fresnel-area ray tracing*. The influence zone about surface wave paths, over which surface waves are coherent in phase, was identified as approximately 1/3 of the width of the first Fresnel zone, representing a sampling region for surface waves at finite frequency. It is, therefore, useful for treating the finite frequency effects of surface wave propagation efficiently in an approximate manner. Furthermore, the use of the Fresnel-area ray tracing allows us to accommodate off-great-circle propagation of surface wave paths in the construction of phase speed maps.

Utilizing these techniques, a new concept of a three-stage approach for surface wave tomography was developed in chapter 5. Subsequently the three-stage inversion was applied to the Australian region in chapter 6, and several sets of 3-D shear wave speed models were constructed. This approach is helpful to compensate for some weaknesses in the conventional methods of surface wave tomography, in that we can incorporate the effects 166 of off-great-circle propagation as well as the finite-frequency of surface waves in frequency and mode domains.

The 3-D models, which are obtained using the influence zone, are naturally smoothed through the finite frequency effects and share similar features of the shear wavespeed variations irrespective of the inclusion of the off-great-circle propagation. This implies that the finite-frequency effects have a more important influence on the tomography models than the effects of ray-path bending in the frequency range used in this study. It should be noted that we have chosen somewhat conservative frequency ranges to avoid the complexity of the short-period surface waves (less than 30 seconds) that are likely to be messed up by the strong heterogeneities in the shallower layers, and thus the deviation of the ray paths from the great-circle are not conspicuous in such frequency ranges.

In chapter 7, we investigated more rigorous two-dimensional sensitivity kernels for phase speed structures of surface waves to take account of the effects of scattering and diffraction with first-order scattering theory. Such an approach will be of importance to perform tomographic inversion considering effects of scattered waves come from outside the influence zone. Although these kernels are preferable to consider more complex phenomena, they require more computational resources.

8.2 Prospects for the future studies on surface wave tomography

The three-stage approach developed and applied in this thesis provides us with a means to incorporate various effects of surface wave propagation efficiently with reasonable approximations. The major objective of the practical application of the three-stage approach in chapter 6 was to assure the utility of the method and to clarify how the inclusion of various phenomena, such as the off-great-circle propagation and the finite-frequency effects, improve the tomography models. The new Australian 3-D models contain expected features, that is, the lateral velocity variations are smoothed naturally by the influence zone. However, some aspects of the technique, which will also be of importance for revealing the mantle structure from surface wave information, are left to future studies. Now we discuss such points that can be developed mainly within a framework of the three-stage inversion.

8.2.1 Inversions for anisotropy

8.2.1.1 Azimuthal anisotropy

Anisotropy in the mantle, which has not been considered in this thesis, provide a crucial information on the structure and dynamics of the current status and the history of the mantle. A number of former studies in both global and regional scale suggested the existence of the azimuthal anisotropy in the upper mantle (e.g., Tanimoto & Anderson, 1985, Montagner & Tanimoto, 1990, 1991; Lévêque et al., 1998; Debayle & Kennett, 2000a). With the use of finite frequency kernels, we can expect that the azimuthal anisotropy, which can be revealed through the phase speed inversion considering the influence zone, will also be affected by the smoothing effects of finite frequency similarly to the smoothing of velocity structure shown in chapter 6.

8.2.1.2 Polarization anisotropy

In chapter 6, we have only worked with Rayleigh wave observations to constrain the isotropic shear wavespeed models, for the preliminary application of the three-stage approach. Although higher-mode phase speed measurements for Love waves tend to be more ambiguous compared to Rayleigh waves due mainly to the significant overlap of the different modes and a low signal-to-noise ratio in horizontal components, path-specific 1-D models derived from appropriate waveform fitting for Love waves can also give satisfactory results of multi-mode phase speeds for the first few modes, as we have seen in chapter 3.

By treating Rayleigh and Love waves independently, we can exploit the possible maximum path coverage for both types of waves, which will be essential to obtain high resolution 3-D SV and SH velocity models. Three-dimensional distribution of polarization anisotropy can then be estimated from these independent observations for SV and SHstructures, which will be of significance to discuss the composition of the Earth's mantle.

8.2.2 Additional constraints

8.2.2.1 Polarization anomaly

For the fundamental-mode with appropriate path length (longer than 50°), we can also make some observations of the arrival angle anomalies of Love and Rayleigh waves using three-component seismograms. Such polarization information is particularly helpful to improve the tomography models since they are sensitive to the lateral velocity gradient perpendicular to the paths, and thus have more sensitivity to the smaller-scale heterogeneity than phase information. Polarization information has previously been applied to the global scale tomography for surface wave phase speed structures (Laske & Masters, 1996; Yoshizawa et al., 1999) and thus these observations can readily be incorporated in the framework of the three-stage approach.

Information from polarization (or arrival angle) anomalies can play an important role in determining the scale of velocity perturbations since they are sensitive to the velocity gradient along a ray, unlike a phase anomaly which depends just on the velocity. As long as we use only phase information, the velocity perturbation depends mainly on the damping applied during inversions. On the contrary, polarization information can be a strong constraint on the size of velocity perturbation. Therefore, by employing the polarization anomalies, we can expect that ambiguities in the maximum and minimum velocity perturbation can be reduced and, eventually, the reliability of the tomography models will be increased. The information of polarization anomaly is not only useful for recovering the velocity structure, but also for the azimuthal anisotropy (Larson et al., 1998; Laske & Masters, 1998).

8.2.2.2 Group speed

The use of phase speed depends on the knowledge of the source mechanisms, which is crucial for the accuracy of the measured phase speed. On the contrary, group speed measurements do not require the source mechanisms and are promising to obtain more stable and reliable measurements (e.g., Levshin et al., 1992; Ritzwoller & Levshin, 1998), although such measurements can only be readily applied to the fundamental mode. There is a group velocity component in waveform fitting, but explicit measurements can extend the frequency range. Group speed information has different sensitivities to the depth from the phase speed, that is, group speed is more sensitive to the shallower structures in the crust and the uppermost mantle than phase speed at the same frequency. Therefore, there are advantages in working with both the phase and group speed information simultaneously to constrain the final 3-D shear wavespeed models.

8.2.3 Toward higher frequency

8.2.3.1 Effects of mode-branch coupling

Since we work with phase speed maps without considering coupling between mode branches in the three-stage approach, we have chosen somewhat conservative frequency ranges over which the assumption of independent mode propagation can be justified and so we can avoid the complex phenomena of the wave propagation in short periods (less than 40 seconds).

However, if there is strong heterogeneity around a ray path, the individual modes of surface waves will not propagate independently and we need to take the coupling between these modes into account. By using 2-D Earth models along a path, Kennett & Nolet (1990) showed that interactions between different modes need to be considered for surface waves with period less than 50 seconds.

The single scattering theory used in chapter 4 and 7 is able to treat just the effects of scattering for the same mode-branch and we cannot consider the mode conversions between different mode branches. One of the ways to tackle such problems has been proposed by Tanimoto (1990b) working with surface wave potential in phase speed domain. A kind of sensitivity kernels considering the effects of mode coupling can be obtained with this technique, although including the effects of coupling between a number of mode branches places considerable computational demands when working, e.g., with a finite difference method, for the computation of the surface wave potentials as adopted in the original form of the technique (Tanimoto, 1990b). Working with reasonable approximations, e.g., restricting the number of modes to be coupled, and the use of the first-order asymptotic results for the surface wave potentials as used in chapter 7, it should be possible to apply in practical inversions of surface waves.

For a 3-D structure, Kennett (1998a) has considered a full 3-D mode-coupling technique extending the idea of 2-D mode coupling along a path (Kennett, 1984), however, the treatment of mode-branch coupling in a 3-D structure is still too complex for the practical use, because we need to consider different directions of propagations for all the scattered waves in a 3-D structure. As long as we can ignore the coupling between different mode types, that is, coupling between Love and Rayleigh waves, the 2-D mode-coupling technique is still helpful in that we can consider the full mode interaction in 2-D structures along the ray path around which most of the surface wave energy are likely to be confined.

Using the concept of the influence zone, we may restrict such areas in which the significant effects of mode coupling can be expected, e.g., the overlapped areas of the influence zone for several modes.

8.2.3.2 Crustal corrections

To push the frequency ranges of interest to much higher levels than those used in this thesis, a careful treatment of the effects of shallower structure, especially the crustal structure, is also essential to avoid undesirable effects on the tomography models. We have used the 3SMAC model (Nataf & Ricard, 1996) to correct the crustal part of the 1-D reference models throughout this thesis, which has worked quite well in the frequency ranges used in this study.

When we use much higher frequency surface waves (up to 20 seconds periods), an appropriate crustal model is critical not only for the inversion for the structure, but also for the forward modeling of surface waves at higher frequency for which the coupling between mode branches should also be taken into account.

Appendix A

Paraxial ray approximation in a ray centered coordinate system

In this appendix, we will derive an explicit formulation for a second partial derivative of the phase ψ in (4.15) and (4.16) in chapter 4 based on the paraxial ray approximation. We first consider the Taylor expansion of the phase ψ around a point on the ray (s, 0) at fixed s,

$$\psi(s,n) = \psi(s,0) + n \frac{\partial \psi(s,n)}{\partial n} \Big|_{n=0} + \frac{1}{2} n^2 \frac{\partial^2 \psi(s,n)}{\partial n^2} \Big|_{n=0}.$$
(A.1)

Since the wavefront is perpendicular to the ray in ray centered coordinates,

$$\frac{\partial \psi(s,n)}{\partial n}\Big|_{n=0} = 0, \tag{A.2}$$

and so the phase ψ at (s, n) can be expressed as,

$$\psi(s,n) = \psi(s,0) + \frac{1}{2}n^2 M(s), \tag{A.3}$$

where $M(s) = \partial_n^2 \psi(s, n)|_{n=0}$.

Following Yomogida (1988), M(s) can be determined from the geometrical spreading evaluated at (s, 0). For the time being, let us introduce a "ray" coordinate system (s, m)(Fig A.1). This coordinate system is different from the "ray-centered" coordinate system. In ray-centered coordinates, the central ray is fixed and a neighbouring point (s, n) is represented by a perpendicular distance n from a point (s, 0) on the central ray (Fig 4.3), whilst in ray coordinates, s is measured along a different ray path passing through the point (s, m) which is under consideration. In other words, we need to consider different coordinates for points on different ray paths as seen in Fig A.1. For the points on the central ray in a ray-centered coordinate system, the "ray" (s, m) and "ray-centered" (s, n)coordinates will be the same.

In the ray coordinates for 2-D case, the Laplacian of the phase ψ can be written as,


Fig. A.1. Ray coordinate system. (s, m) is evaluated from a ray passing through that point and another orthogonal coordinates (s', m') is introduced for all rays. So that the direction of m is always perpendicular the ray and tangent to the wavefront.

$$\nabla^2 \psi = \frac{1}{h_1 h_2} \left[\frac{\partial}{\partial s} \left(\frac{h_2}{h_1} \frac{\partial \psi}{\partial s} \right) + \frac{\partial}{\partial m} \left(\frac{h_1}{h_2} \frac{\partial \psi}{\partial m} \right) \right],\tag{A.4}$$

where h_1 and h_2 are scaling factors for s and m, respectively. In this ray-coordinate scheme, s is always a tangent to the ray path and m is perpendicular to the ray path, resulting in,

$$\frac{\partial \psi}{\partial s} = k = \frac{\omega}{c}, \quad \frac{\partial \psi}{\partial m} = 0.$$
 (A.5)

Substituting (A.5) into (A.4) with the scaling factors $h_1 = 1, h_2 = J$,

$$\nabla^2 \psi = \frac{\omega}{J} \left[\frac{\partial}{\partial s} \left(\frac{J}{c} \right) \right] = \frac{\partial}{\partial s} \left(\frac{\omega}{c} \right) + \frac{\omega}{cJ} \frac{dJ}{ds}, \tag{A.6}$$

where the constant J is assumed to be a function of s. The scaling factor $h_2 = J$ corresponds to the geometrical spreading.

Next let us get back to ray-centered coordinates (s, n) and investigate the relation between M and J. By differentiating (A.3) with respect to s and n,

$$\frac{\partial \psi}{\partial s} = k + \frac{1}{2}n^2 \frac{dM}{ds},\tag{A.7}$$

$$\frac{\partial \psi}{\partial n} = Mn. \tag{A.8}$$

The Laplacian of the phase ψ in this case can be obtained from (A.7) and (A.8),

Paraxial ray approximation in a ray centered coordinate system

$$\nabla^2 \psi|_{n=0} = \left[\frac{\partial^2 \psi}{\partial s^2} + \frac{\partial^2 \psi}{\partial n^2}\right]_{n=0} = \frac{\partial}{\partial s} \left(\frac{\omega}{c}\right) + M(s).$$
(A.9)

We then get the relation between M and J by comparing (A.6) and (A.9),

$$M(s) = \frac{\omega}{c(s)J(s)} \frac{dJ(s)}{ds}.$$
(A.10)

Since we already know the geometrical spreading J as in (4.13), the evaluation of M(s) is straightforward.

In (A.10), we need to differentiate J with respect to s. This can be done analytically using (4.13) and the derivative can be expressed in terms of the solutions of the DRT equations (4.6)-(4.8),

$$\frac{dJ}{ds} = \frac{1}{J} \left[-\sin\zeta \frac{\partial\theta}{\partial\zeta_0} \frac{\partial\zeta}{\partial\zeta_0} + \sin\theta \frac{\partial\phi}{\partial\zeta_0} \left(\cos\theta \cos\zeta \frac{\partial\phi}{\partial\zeta_0} - \cot\theta \sin\zeta \frac{\partial\theta}{\partial\zeta_0} + \cos\zeta \frac{\partial\zeta}{\partial\zeta_0} \right) \right].$$
(A.11)

Appendix B

Correction of the paraxial Fresnel area at source and receiver

The radius of the paraxial Fresnel area in (4.23) in chapter 4 shrinks to 0 at the source and receiver locations because the geometrical spreading J disappears there. However, as seen in Fig 4.6, the exact Fresnel area should have finite radius even at such singular points. Correction of the paraxial Fresnel area can be made by considering the simple geometry around source and receiver (Fig B.1).

We pay attention only to the source region as shown in Fig B.1. To extend the paraxial Fresnel area around the source location, it is sufficient to consider two points F_a and F_b . Let us consider a path AF_aB ; in the far field, we can expect $\Delta_A^{F_a} \ll \Delta_A^B$, so that $\Delta_A^B \approx \Delta_B^{F_a}$. Using (4.24), the radius of the paraxial Fresnel area $\Delta_A^{F_a}$ at the source can be given as,

$$\Delta_A^{F_a} \approx \frac{\lambda}{2}.\tag{B.1}$$

For a path AF_bB along the ray, we can extend the paraxial Fresnel area slightly over the source position. In this case, $\Delta_B^F = \Delta_A^F + \Delta_A^B$. Inserting this relation into (4.24),

$$\Delta_A^{F_b} = \frac{\lambda}{4}.\tag{B.2}$$

The location of the point F_b relative to the source can be obtained from an extrapolation along the ray.

The extended paraxial Fresnel area can be obtained from interpolation of such points surrounding the source on the boundary of the first Fresnel zone. The corrections for the receiver and caustics can be similarly made by following the same procedure as above. The correction of the influence zone at these points is obtained in a similar fashion; we can evaluate the radius of the zone to be 1/3 of that of the first Fresnel zone, i.e., $\Delta_A^{F_a} \sim \lambda/6$ and $\Delta_A^{F_b} \sim \lambda/12$



Fig. B.1. Illustration of the correction of the paraxial Fresnel area at source and receiver.

Appendix C

Model parameterisation for phase speed maps

In this appendix, we explain the parameterisation of phase speed models used in chapter 6. In order to represent the phase speed distribution in a two-dimensional parameter space (θ, ϕ) , we first divide the region of interest into a set of the geographic cells, and then consider knots at the centre of each cell (Fig C.1 (a)). We do not simply use the phase velocity in these cells as model parameters, but we use a cubic B-spline parameterisation on a sphere (e.g., Wang & Dahlen, 1995b) to smooth the models. The cubic B-splines centred at these knots are represented by

$$\mathcal{F}(\mathcal{D}) = \begin{cases} \frac{3}{4}\mathcal{D}_1^3 - \frac{6}{4}\mathcal{D}_1^2 + 1 & : \mathcal{D} \le \bar{\mathcal{D}}, \\ -\frac{1}{4}\mathcal{D}_2^3 + \frac{3}{4}\mathcal{D}_2^2 - \frac{3}{4}\mathcal{D}_2 + \frac{1}{4}, & : \bar{\mathcal{D}} \le \mathcal{D} \le 2\bar{\mathcal{D}}, \\ 0 & : \mathcal{D} \ge 2\bar{\mathcal{D}} \end{cases}$$
(C.1)

where $\bar{\mathcal{D}}$ is the average distance between neighbouring knots, and $\mathcal{D}_1 = \mathcal{D}/\bar{\mathcal{D}}$ and $\mathcal{D}_2 = (\mathcal{D} - \bar{\mathcal{D}})/\bar{\mathcal{D}}$.

Using such spherical B-splines as the basis function, the phase speed variation at an arbitrary point (θ, ϕ) on a sphere can be represented as,

$$\frac{\delta c(\theta,\phi)}{c} = \sum_{j=1}^{N} m_j \mathcal{F}_j(\mathcal{D}(\theta,\phi)), \tag{C.2}$$

where N is the total number of the model parameters, and $\mathcal{D}(\theta, \phi)$ is the distance between the point (θ, ϕ) and the knot at the centre of the *j*th basis function (Fig C.1 (b)). An example of the spherical spline function $\mathcal{F}(\mathcal{D})$ is shown in Fig C.2 (a).

The average interval \mathcal{D} between knots controls the horizontal smoothness of the phase speed model. In this study, the use of the dense coverage of paths (Fig 6.1 (b)) enables us to set $\overline{\mathcal{D}}$ to be as small as 2°.



Fig. C.1. (a) Illustration of the cells divided by the geographic longitudinal and latitudinal lines. At the centre of each cell, we define the knots for which the spherical spline functions are defined. (b) Geometrical configuration for \mathcal{D} and $\bar{\mathcal{D}}$ around the *j*th cell. Note that the average distance $\bar{\mathcal{D}}$ need not be exactly the distance between two neighbouring knots as shown in this figure.

When the geographical cells are used as a guide to parameterise the 2-D space on a sphere, we need to take care of the varying sizes of the cells with latitude. We have checked the effects on the final models through several tests working with different measures for $\bar{\mathcal{D}}$ based either on the apparent geographical distance or on the actual geodetic distance (Fig C.2).

For the geographic distance, $\overline{\mathcal{D}}$ is fixed and measured as $2^{\circ} (\approx 222 \text{km})$ everywhere on the sphere. On the other hand, for the geodetic distance, $\overline{\mathcal{D}}$ varies with latitude and the azimuth from a knot location to a target position (θ, ϕ) , e.g., for a knot at the equator, $\overline{\mathcal{D}}$ is always $2^{\circ} (\approx 222 \text{ km})$ for all directions, and for a knot at -31° latitude, $\overline{\mathcal{D}} = 2^{\circ} \approx 190$ km along a parallel of latitude whereas $\overline{\mathcal{D}} = 2^{\circ} \approx 222$ km along a meridian.

Examples of the cubic B-splines are shown in Fig C.2. Near the equator, there is almost no distortion due to the variation of distance measured either as the geodetic (Fig C.2 (b)) or as the geographic distances (Fig C.2(c)). However, as we go to higher latitudes, there is some distortions caused by the differences in the measures of distance. In Fig C.2 (d), with the geodetic measure for \bar{D} the B-spline takes an egg-like shape due to the shrinkage of the actual geodetic distance at 31° south latitude. If we measure \bar{D} as the geographic distance, the pattern of the B-splines (Fig C.2 (e)) at 31° south latitude is nearly the same as that near the equator (Fig C.2 (c)) showing the similar lateral extent of the B-splines. Thus the region which is covered by one B-spline function in the geographic measure of \bar{D} becomes larger than in the geodetic case. To adjust the contribution from



Fig. C.2. (a) Spherical B-spline function \mathcal{F} at a knot centred upon $(1^{\circ}, 1^{\circ})$ with average knot interval $\overline{\mathcal{D}} = 2^{\circ}$. $\overline{\mathcal{D}}$ is measured as the geodetic distance. (b) Same as (a) but viewed from a different angle. Green dots show locations of knots. (c) Spherical B-spline function at the same location as (b) but the distance $\overline{\mathcal{D}}$ is measured as the geographic distance. (d) Same as (b) but the knot is centred at $(-31^{\circ}, 1^{\circ})$. (e) Same as (c) at $(-31^{\circ}, 1^{\circ})$. For the geographic measure of $\overline{\mathcal{D}}$ in (c) and (e), the \mathcal{F} is multiplied by $\sin(\Theta)$, where Θ is the colatitude of the knot position.

a single basis function located in regions where there are high density of knots (i.e., at higher latitude), we multiply \mathcal{F} by a term $|\sin(\Theta)|$ depending on the colatitude Θ when we use the geographic measure of $\overline{\mathcal{D}}$. Therefore, the amplitude of the B-spline is somewhat suppressed in Fig C.2 (e).

We have applied both of the schemes for measuring $\overline{\mathcal{D}}$ in the actual inversions for phase speed maps. Despite the differences in the definition of $\overline{\mathcal{D}}$, the models obtained are almost identical and do not show any conspicuous differences. Therefore, in this study, we adopt the geodetic measure for $\overline{\mathcal{D}}$.

Although our tests with different ways of measuring $\overline{\mathcal{D}}$ suggest that the choice of the geographic or the geodetic distances does not have a significant impact on the final results, the configuration of the knots in this study is still based on the geographical cells. Thus we should be careful about the possible bias in the regions which are far from the equator where the densities of the knots become higher. For global studies and regional studies including polar regions, the use of a triangular-type representation with a comparable size for all the cells on a sphere (e.g., Wang & Dahlen, 1995b, van der Lee & Nolet, 1997) will be helpful to compensate for the deficiencies in geographical grids, although this is not the case in this study for which the region of major interest lies in a comparatively narrow zone between 10° and 40° south latitude.

Appendix D

Rayleigh-wave phase speed maps

In this appendix, Rayleigh wave phase speed maps for the fundamental and the first three higher modes used to constrain the 3-D shear wave speed model for a model set *Riz-GCiz* are displayed at every 20 seconds.



Fig. D.1. Phase speed maps *Riz-GCiz* for fundamental-mode Rayleigh waves.



Fig. D.2. Phase speed maps *Riz-GCiz* for 1st higher-mode Rayleigh waves.



Fig. D.3. Phase speed maps *Riz-GCiz* for 2nd higher-mode Rayleigh waves.



Fig. D.4. Phase speed maps *Riz-GCiz* for 3rd higher-mode Rayleigh waves.

Appendix E

3-D shear wavespeed model gallery

The full sets of 3-D shear wavespeed models and corresponding cross sections for all types of models (GC0, RAY-GC0, GCiz, Riz-GC0 and Riz-GCiz) obtained in chapter 6 are displayed in this appendix. The relation of these models are summarised in Fig 6.7.



Fig. E.1. Shear wavespeed model GC0 in the upper mantle. Reference velocities are 4.41 km/s at 100 km, 4.43 km/s at 150 km, 4.51 km/s at 200 km, 4.61 km/s at 250 km, 4.70 km/s at 300 km and 4.75 km/s at 350 km.



Fig. E.2. Cross sections of shear wave speed model $GC\theta$ in Fig E.1.



Fig. E.3. Shear wavespeed model RAY-GC0 in the upper mantle. Reference velocities are the same as Fig E.1



Fig. E.4. Cross sections of shear wave speed model RAY-GC0 in Fig E.3.



Fig. E.5. Shear wavespeed model GCiz in the upper mantle. Reference velocities are the same as Fig E.1



Fig. E.6. Cross sections of shear wave speed model *GCiz* in Fig E.5.



Fig. E.7. Shear wavespeed model $Riz\text{-}GC\theta$ in the upper mantle. Reference velocities are the same as Fig E.1



Fig. E.8. Cross sections of shear wave speed model *Riz-GC0* in Fig E.7.



Fig. E.9. Shear wavespeed model $Riz\mathcar{-}GCiz$ in the upper mantle. Reference velocities are the same as Fig E.1



Fig. E.10. Cross sections of shear wave speed model *Riz-GCiz* in Fig E.9.

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